Voronoi-based 9-Intersection Model for Topological Spatial Relations

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Abstract

Spatial relations is a key component of Geographical information science (GIS). Efforts have been made on formal definition of spatial relations and the 9-intersection model developed by Egenhofer and his collaborators is the most popular framework for the topological spatial relations. In that model, the topological relationships between two simple spatial entities A and B is transformed into point-set topology problem in terms of the intersections of A's boundary, interior, and exterior with B's boundary, interior and exterior. The exterior of an entity is then defined as its complement. There are some shortcomings with this model both in theory and in practice. This paper presents a modified version of the 9-intersection model. The modification is made by replacing the complement (exterior) of an entity with its Voronoi region and the resulting new model is called Voronoi-based 9-intersection model (V9I). It is also demonstrated that the shortcomings associated with original 9-intersection model are overcome by the modified model -- V9I.

1. Introduction

Spatial reasoning is a major requirement for a comprehensive GIS (Frank, 1991) because it offers users new spatial information, which has not been explicitly recorded and which is otherwise not immediately available in the form of raw data (Egenhofer, 1994). To facilitate such reasoning, spatial relations between entities have to be established. In this sense, some researchers even argue that spatial relations between spatial entities are as important as the entities themselves.

Over the past two decades, research has been conducted of how to apply fundamental mathematical theories for modeling and describing spatial relationships (Peuquet 1986, Jungert 1988, Chang et al. 1989, Lee and Hsu 1990, Egenhofer and Franzosa 1991, Egenhofer and Al-Taha 1992, Smith and Park 1992, Cui et ac 1993, Kainz et al 1993). Here, no attempt has been made to discuss various types of spatial relations. Instead, this paper concentrates on topological relations as "topological properties are most fundamental, compared to Euclidean, metric and vector spaces" (Egenhofer, 1991).

Topological relations are those which are invariant under topological transformation. That is, they are preserved if the entities are translated, rotated or scaled (Egenhofer, 1991). A formalized representation of topological relations has been developed based on point-set topology (Guting 1988, Pullar 1988, Egenhofer 1989). The results of such a formalism are the so-called 4-intersection and 9-intersection models (Egenhofer and Franzosa 1991). Indeed, the later is an improvement of the former (Egenhofer *et al.*, 1993). In these models, the topological relations between two entities A and B are defined in terms of the intersections of A's boundary, interior and exterior with B's boundary, interior and exterior. The exterior of an entity is then represented by its complement.

The 9-intersection model is the most comprehensive model for topological spatial relation so far. It has been used or extended for examining the possible topological relations between regions in discrete space [Egenhofer and Sharma,1993; Winter, 1995], modeling conceptual neighborhoods of topological line-region relations [Egenhofer and Mark, 1995], grouping the very large number of different topological relationships for point, line and area features into a small sets of meaningful relations [Clementini *et al.,* 1993], describing the directional relationships between arbitrary shapes and flow direction relationships [Abdelmoty and Williams, 1994; Papadias and Theodoridis, 1997], deriving the composition of two binary topological relations [Egenhofer,1991], describing changes to topological relationships by introducing a Closest-Topological-Relationship-Graph and the concept of a topological distance [Egenhofer and Al-Taha, 1992], analyzing the distribution of topological relations in geographic data sets [Florence III and Egenhofer, 1996], as well as formalizing the spatio-temporal relations between the father-son parcels during the process of land subdivision [Chang and Chen, 1997]. These investigations have significantly contributed to the development of the state-of-art spatial data models and spatial query functionality [Egenhofer and Mark, 1995; Mark et al., 1995; Papadis and Theodoridis, 1997].

However, as will be discussed later in Section 2, even the improved 9-intersection model have problems both in theory and in practice. Examples for the former are the difficulties in distinguishing different disjoint relations and relations between complex entities with holes. Example for the latter is the difficulty or impossibility in computing the intersections with an entity's complement since the complements are infinitive. To improve this situation, this paper presents a modified model, called Voronoi-based 9-intersection model, which is a result of replacing the complements of spatial entities by their Voronoi regions.

Following this introduction is a review and analysis of the existing 9-intersection model. In this section (Section 2), the problems with this model will be examined and the modified 9-intersection model, Voronoi-based 9-intersection model, will be presented (Section 3). The possible topological relations using the modified model are discussed (Section 4). Finally, some future research directions are pointed out (Section 5).

2. A critical examination of the 9-intersection model

In order to present an improved model in the next section, it seems logic to conduct a critical examination of the existing 9-intersection model in this section to see what kind shortcomings it possesses and what kind of improvement can be made.

2.1 A review of the 9-intersection model

In the early stage of research, the so-called 4-intersection model for topological relations was proposed by (Munkres, 1975; Egenhofer, 1989) based on pointset topology. The principle is as follows: Suppose A and B are two sets representing two entities, then the spatial relations between A and B can be described by all possible true values for the 4-tuples as follows:

$$R_4(A,B)=[\partial A\cap\partial B,\ A^0\cap\partial B,\ \partial A\cap B^0,\ A^0\cap B^0] \qquad (1)$$

where ∂A is the boundary of A and A^0 is the interior of A and the annotation for B is the same. For example, if A and B are disjoint, then the values for these 4-tuples are [ϕ, ϕ, ϕ, ϕ]. For another example, if A and B are overlapping, then the 4 values becomes [$-\phi$, $-\phi$, $-\phi$, $-\phi$]. Here ϕ means empty and "$-\phi$" means non-empty. These relations are mutually exclusive and form a partition of the set of all relations such as "disjoint", "overlap", "touch", "equals", "cover", "in", etc.

However, as pointed out by Clementini *et al.* (1993), there are some cases where some confusions may be caused by this 4-intersection model. For example, the 4-intersection model will have difficulty in judging whether or not an entity is completely included in another one. Figure 1 shows such examples. For this reason, Egenhofer and Franzosa (1991) have made an extension to this 4-intersection model, leading to a new model, called 9-intersection model, as follows:

$$R_9(A,B) = \begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^o & \partial A \cap B^- \\ A^o \cap \partial B & A^o \cap B^o & A^o \cap B^- \\ A^- \cap \partial B & A^- \cap B^o & A^- \cap B^- \end{bmatrix} \quad (2)$$

Here, ∂A, A^o, A^- mean the boundary, interior and exterior of A, respectively. The annotation for B is the same. In this model, the exterior of A is normally defined as the complement of A. It is clear, as shown in Figure 1, some of the shortcomings associated with the 4-intersection model can be overcome by this model.

Figure 1 Improvement of the 9-intersection over 4-intersection model.

(For the first two relations, the 4-intersection model fails to distinguish between them, however, in the 9-intersection model, the distinction is clear. The situation is similar for the last 3 relations)

The 9-intersection model is the most popular mathematical framework for formalizing topological spatial relations. In this model, by considering whether the value (i.e. empty or non-empty) of the 9-intersections, a range of binary topological relations could be identified [Egenhofer and Franzosa, 1991]. For instance, eight relations, as shown in Figure 2a, can be identified between two spatial regions in R^2, i.e. *disjoint*, *meet*, *equal*, *inside*, *contains*, *covers*, *covered-by* and *overlap* [Egenhofer and Sharma,1993]. Similarly, as shown in Figure 2, topological relations between area-line, area-point, line-line, line-point as well as point-point can also be defined [Egenhofer and Herring, 1991; Egenhofer, 1993; Sun *et al*., 1993].

Figure 2 Topological spatial relations defined by the 9-intersection model

(a) Between two areas; (b) Between area and line; (c) Between area and point; (d) Between two lines; and (e) Between line and point

These relations are mutually exclusive. That is, only one of them holds at a time for any particular configuration.

2.2 Shortcomings of the 9-intersection model

Although the 9-intersection model is the most comprehensive model for topological spatial relations so far, yet it does possess some shortcomings.

It was found that the number of relations existing among entities depends on the dimension of the space with respect to the dimension of the entities and on topological properties of the entities embedded in that space [Egenhofer and Sharma,1993]. In this respect, a dimension extended method was proposed by Clementini *et al*. (1993) to take into consideration the dimension of intersection results in addition to the values (i.e. empty or non-empty). The dimension extended method was also used for formalizing topological relations in 3D space [Guo and Chen, 1997].

More seriously, there are some fundamental shortcomings associated with this model as follows:

1. It fails to distinguish certain disjoin relations, as shown in Figure 3;
2. It fails to identify the topological relations between two regions with holes as shown in Figure 4. Indeed, this model can only deal with simple entities as the homogeneous, 2-dimensional and connected areas, and, lines with exactly two end points [Egenhofer,1993]. If there a hole in a region, each of these regions should be separated into its generalized region and its holes, and the combinatorial intersections of the generalized regions and holes would then be examined [Egenhofer *et al*., 1994].
3. It is quite computationally intensive or even impossible to compute the intersections with an entity's complement.

As a consequence, in the SDTS, compound spatial relations are defined with aggregates of two or more 9-intersection primitives [*Mark et al*., 1995], but it is still difficult to calculate these primitives from spatial data.

Then one may ask "What are the causes of these problems? Is there any solution?" The answer to the latter is "yes". In the next section, the causes of these problems will be discussed and a solution provided.

3. Voronoi-based 9-intersection model: An improved solution

As has been discussed in the previous section, there are some fundamental shortcomings with the 9-intersection model. This section tries to find out the causes and to solve these problems.

3.1 The causes of the problems associated with 9-intersection model

The third problem -- intensive computation -- can be easily understood because the exterior of a spatial entity is defined by its complement. The complement of an entity is the set of all points of R^2 not contained in that entity [Egenhofer, 1991]. Therefore, it is infinitely large. Furthermore, there are five complement-related intersections out of the 9 in this model, i.e. $\partial A \cap B^-$, $A^0 \cap B^-$, $A^- \cap \partial B$, $A^- B^0$, and $A^- \cap B^-$. That is, this problem is due to the *infiniteness of the complement* and it can be solved by replacing the complement with a small region surrounding the entity.

The first and the second problems - failure to differentiate certain relations -- are more serious. It means that the model is not a generic one. Failure to differentiate two relations implies that the values for the elements of the 9-intersection model remain unchanged even though the actual relation become different. Then, why?

In the case of disjoint as shown in Figure 3, the values of the 9-intersections for Figure 3b and 3c remain the same as these for Figure 3a although there is an additional entity between A and B. This is due to the fact that the infiniteness of a complement guarantee

(a) an overlap between the complements of A and B;

(b) A's complement always intersects with the boundaries, interiors and complements of entity B if there are disjoint regardless the existence of additional features in between, and vice versa. Therefore, the five complement-related elements always take the non-empty value ($-\varnothing$).

This problem can be solved only if the two exteriors of A and B are mutually exclusive when they are disjoint.

------------**I don't quite understand the hole cases, please clarify them**-------------

In the case of complex entities with holes as shown in Figure 4, th. For the cover and cover-by relations between A and B in Fig. 6a, two distinct 9-intesections can be found. However, the 9-intersection would be the same when A or B has a hole, as shown in Fig.6b. The interior and boundary of the entity falling in the hole (i.e., A) do not intersect the interior of the other entity (i.e., B), so $\partial A \cap B^0 = \emptyset$, $A^0 \cap B^0 = \emptyset$, and $A^0 \cap \partial B = \emptyset$. Moreover, the five complement-related intersections $\partial A \cap B^-$, $A^0 \cap B^-$, $A^- \cap \partial B$, $A^- \cap B^0$ and $A^- \cap B^-$ all take non-empty values. Some other examples are illustrated in Fig.6c, 6d and 6e.

[Fig.4 Problems caused when the area entity with holes]

3.2 The Voronoi-based 9-intersection model: the solution

As discussed in the previous section, to make the 9-intersection model become a generic model, the following two requirements for the exterior must be met:

(a) The exterior of an entity should be as small as possible; and

(b) The exteriors of two disjoint entities must be exclusive.

As a result of this reasoning, the Voronoi region as shown in Figure 5 seems to be the "if and only if" candidate for the exterior of an spatial entity because the Voronoi region of an entity has a special meaning - the 'influence region' of itself and is defined as the area containing all locations closer to itself than to any other. Suppose we have a set of spatial entities, $SO=\{O_1,\ldots, O_i ,\ldots, O_n\}$ $\{1 \leq n \leq \infty \}$, in IR^2, O_i may be a point entity O_P, or line entity O_L or area entity O_A. An area entity is not necessarily convex, and may have holes in which another area may exist. The Voronoi region of O_i (called O^v) can be mathematically defined as follows:

$$V(O_i)=\{p|ds(p, O_i) \leq ds(p,O_j), j\neq i, i, j\in In \} \qquad (3)$$

Indeed, Voronoi-based tessellation - Voronoi diagram - is closer to human perceptions, poses a variety of challenges to the 'usual way of doing things' in GISs [Gold, 1989, 1991,1992; Wright and Goodchild, 1997] and has found wide applications [Yang and Gold, 1994; Gold et al., 1996; Edwards et al., 1996; Hu and Chen, 1996; Chen and Cui, 1997]. However, a detailed discussion of Voronoi diagram itself lies outside of this paper.

[Fig.5 Voronoi diagram of point, line and area entities]

In other word, by replacing the complement of an entity with its Voronoi region, a Voronoi-based 9-Intersection (called V9I briefly) framework can be formulated as the following:

$$R_{V9}(A,B) = \begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^{o} & \partial A \cap B^{V} \\ A^{o} \cap \partial B & A^{o} \cap B^{o} & A^{o} \cap B^{V} \\ A^{V} \cap \partial B & A^{V} \cap B^{o} & A^{V} \cap B^{V} \end{bmatrix} \quad (4)$$

where A^v is Entity A's Voronoi region and B^v is Entity B's Voronoi region. $\partial A \cap B^v$, $A^0 \cap B^v$, $A^v \cap \partial B$, $A^v \cap B^0$ and $A^v \cap B^v$ are the elements related to Voronoi regions and they can be easily computed.

Indeed, this model is capable of overcoming the shortcomings of the original 9-interscetion model, as will be discussed in the next section.

4. Topological relations with V9I

After the model is proposed, it seems necessary (a) to examine the spatial topological relations defined by this model, (b) how the shortcomings of the original 9-intersection model can be overcome, and (c) what kind of addition relations this new model may be able to distinguish.

4.1 Topological relations with V9I

Topological relations between point, line and area entities can be formalized with the new model, including relations between area-area, line-area, area-point, line-line, line-point and point-point entities. The possible results are listed in Table 1. A diagrammatic representation of these relations is given in Figures 6-12.

Among the thirteen topologically distinct relationships between two areas distinguished by the V9I, seven of them, as shown in Figures 6g-6m, could not be distinguished using the original 9-interscetion model.

----- How about area-line, area-point, line-line, line-point & point-point ???----

Table 1 Distinguished topological relationships using V9I

Cases		Number	Names
AA	Area/Area	13	
LL	Line/Line	8	
LA	Line/Area	13	
PP	Point/Point	3	
PL	Point/line	4	
PA	Point/Area	5	

Figure 6 Topological relations for area-area relations

Figure 7 Topological relations for area-line relations

Figure 8 Topological relations for area-point relations

Figure 9 Topological relations for line-line relations

Figure 10 Topological relations for line-point relations

Figure 11 Topological relations for point-point relations

4.2 Resolution of problems with original 9-intersection model

One characteristics of this new model is that $A^v \cap B^v$ would be non-empty when two entities are adjacent, such as A and B in Figure 8a, because the Voronoi region of A shares the same boundary with that of B. However, when there is an entity C between A and B (Figure 8b), their Voronoi regions are separated by that of C and $A^v \cap B^v$ is empty. Therefore, it is easy for this new model to overcome the shortcomings of the original 9-intersection model in distinguishing disjoint spatial entities. Figures 13 shows how these relations between disjoint entities which the original 9-intersection fails to distinguish are distinguished by this new model.

Another characteristic of this new model is that $\partial A \cap \partial B$ and $A^v \cap B^v$ would both take non-empty values when the boundary of entity A meets with that of entity B as their Voronoi regions would also meet according to the definition of Voronoi diagram. An illustration is shown in Figure 9a. In addition, the boundary of entity A meets with B^v and B's boundary meet with A^v. As a result, the relation *meet* between area entities defined by Voronoi-based 9-intersection model is therefore quite different from that by the original 9-intersection. For entity with a hole e.g. B as shown in Figure 9b, if the boundary of the other entity, i.e. A, meets its inner boundary, A's Voronoi region intersects B's inner boundary, resulting in 4 non-empty elements as follows:$\partial A \cap \partial B = -\varnothing$, $A^v \cap \partial B = -\varnothing$, $A^v \cap B^v = -\varnothing$ and $\partial A \cap B^v = -\varnothing$. Moreover, if the whole body of A is contained in the hole of B, it means that A's interior overlaps with the convex of B and $A^0 \cap B^v = -\varnothing$. These characteristics make the Voronoi-based 9-interscection model be capable of distinguishing relations between complex entities with holes.

The example shown in Fig.9b has the same original 9-intersection, but has a different Voronoi-based 9-intersection than Fig.9a. The example illustrated in Fig.9d is a *contained-by* relation which has the same 9-intersection with the *contains* relation shown in Fig.9e. The Voronoi regions touch and there is not intersection of boundaries and interiors between the two entities. However, the boundary and interior of the contained entity intersect with the Voronoi convex of the other entity. Another example is given by Fig. 9f where a line meets a homogeneously 2-dimensional and connected area B and the line falls into an area's hole in Fig.9g.

[Fig.9 Distinguishing relations between complex entities with V9I]

4.3 High-resolution neighbour relations by V9I

Due to the adjacency of neighbouring Voronoi regions, it is also possible to distinguish the adjacent relations from other disjoint relations with the V9I. More adjacent relations could be defined and derived **with the Voronoi diagram (???)**, such as immediate neighbor, nearest neighbor, second-nearest, lateral neighbor, tracing neighbor, etc. [Chen *et al*., 1997]. Figure ** shows such relations.

5. Discussions and conclusions

In this paper, it has been discussed that there are some shortcomings associated with existing 9-intersection model for spatial topological relation, both in theory and in practice. From both theoretical viewpoint, it cannot be used as generic model because fails to distinguish disjoint relations while about 80% spatial relations are disjoint relations [Florence and Egenhofer, 1996]. It fails to distinguish the relations of spatial entities with holes. From practical view point, it is also computation-intensive in practice or almost impossible to compute.

It has been revealed through analysis, these problems are caused by a single reason -- the use of complement for the exterior of an entity. These shortcomings can be overcome if (a) the exterior of an entity sufficiently small; and (b) the exteriors of two disjoint entities are exclusive. And it is reasoned that the Voronoi region of an entity is the “if and only if” choice for the exterior of itself. By using the Voronoi region to replace the complement, a new model, Voronoi-based 9-intersection model, is proposed.

Indeed, discussion in Section 4 demonstrates that the Voronoi-based 9-interscetion is not only capable of solving all these three major problems associated with the original

9-intersection model as discussed previously but also capable of distinguishing more adjacency relations. The

The superiority of Voronoi-based 9-intersection model can also be explained from methodological point of view. That is, this model is a integrated model of the two possible approaches for formalizing spatial relations - intersection-based and interaction-based models as classified by Abdelmoty *et al.* [1994]. In the former, an entity is represented in terms of its components and the relationships are the result of the combinatorial intersection of those components, with the original 9-intersection model being the typical example. In the latter, the body of an entity is considered as a whole and is not decomposed into its components. This claim is evidenced by the fact that the four sets $\partial A \cap \partial B$, $\partial A \cap B^0$, $\partial B \cap A^0$ and $A^0 \cap B^0$ of the V9I take the intersection between entities into account and the interactions between adjacent entities can be distinguished with the five sets $\partial A \cap B^v$, $A^0 \cap B^v$, $A^v \cap \partial B$, $A^v \cap B^0$ and $A^v \cap B^v$ by generating Voronoi region for the whole body of each spatial entity.

At this point, one might have been convinced that this new model is a logic modification of the original 9-intersection model but may still wonder their feasibility of practical use because existing algorithms for the generation of Voronoi diagrams are normally limited for point features. However, this is not an issue anymore. Indeed, both vector and raster approaches have been developed for generating Voronoi diagram of points, lines and areas [Gold and Yang, 1995; Li, *et al*., 1998; Okabe *et al*., 1992;].

It must be pointed out here that spatial relation is still an unsolved problem. More research in this area is very desirable. Indeed, the NCGIA at Maine is also devoting its efforts in this area in the next few year, as understood from its home page on the Web. For the next few years, the authors would like to conduct (a) a comparison of the results of V9I with the original 9-intersection; (b) development of a special toolkit for manipulating spatial relations with V9I; and Inference of spatial relations with the V9I.

Acknowledgments ---------- this might be added later on**.**

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