

# VISUALIZING TERRAIN MODELS FROM CONTOURS – PLAUSIBLE RIDGE, VALLEY AND SLOPE ESTIMATION

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## ABSTRACT:

Applications such as modelling and animation of water runoff are becoming more and more important, and demand better surface orientation information than is available from traditional interpolation techniques and grids. Despite modern data acquisition techniques, contour maps are still the most available form of elevation information. Recent work on the automatic reconstruction of curves from point samples, and the generation of medial axis transforms (skeletons) has greatly helped in expressing the spatial relationships between sets of topographic contours. With these techniques the insertion of skeleton points into a TIN model guarantees the elimination of all "flat triangles" where all three vertices have the same elevation. Additional assumptions about the local uniformity of slopes give us enough information to assign elevation values to these skeleton points. In addition, various interpolation techniques were compared using the enriched contour data. Examination of the quality and consistency of the resulting maps indicates properties of the interpolation method that are required in order to produce terrain models with valid slopes. The result provides us with a surprisingly realistic model of the surface - that is, one that conforms well to our subjective interpretation of what a real landscape should look like, and how it should behave under water runoff and erosion.

## Introduction

This paper concerns the generation of interpolated surfaces from contours. While this topic has been studied for many years (Dayhoff, 1963; Crain, 1970; Davis, 1973; Peucker, 1978; Sibson, 1982; Walters, 1969), the current project is interesting for a variety of reasons. Firstly, contour data remains the most readily available data source. Secondly, valid theorems for the sampling density along the contour lines have only recently been discovered (Amenta *et al.*, 1998). Thirdly, the same publications provide simple methods for generating the medial axis transform, or skeleton, which definitively solves the "flat triangle" problem (which often occurs when triangulating contour data) by inserting additional points from this skeleton. Fourthly, the problem of assigning elevation values to these additional ridge or valley points can be resolved, using the geometric properties of this skeleton, in ways that may be associated with the geomorphological form of the landscape. In addition, comparisons of the methods used in a variety of weighted-average techniques throw light on the key components of a good weighted-average interpolation method, using three-dimensional visualization tools to identify what should be "good" results – with particular emphasis being placed on reasonable slope values, and slope continuity. This last is often of more importance than the elevation itself, as many issues of runoff, slope stability and vegetation are dependent on slope and aspect – but unfortunately most interpolation methods cannot claim satisfactory results for these properties.

## GEOMETRIC PRELIMINARIES

The methods discussed here depend on a few fundamental geometrical constructs that are now fairly well known – the Voronoi diagram and its dual, the Delaunay triangulation, as shown in Fig. 1. The first is often used to partition a map into

regions closest to each generating point; the second is usually used as the basis for triangulating a set of data points, as it is guaranteed to be locally stable. The Delaunay triangulation may easily be constructed using its "empty circumcircle" property – this circle is centred at the Voronoi node associated with each triangle.



Figure 1. Delaunay triangulation and Voronoi diagram

The Voronoi diagram and Delaunay triangulation are associated with other geometric structures, known as the crust and the skeleton (or "medial axis transform"), using algorithms introduced by (Amenta *et al.*, 1998). They examined the case where a set of points sampled from a curve or polygon boundary were triangulated, and then attempted to reconstruct the curve. They showed that this "crust" was formed from the triangle edges that did not cross the skeleton, and that if the sampling of the curve was less than 0.25 of the distance to the skeleton formed by the remaining Voronoi edges the crust was guaranteed to be correct. Fig. 2 shows the triangulation of several points: the crust is marked with thick black lines and the skeleton with thick grey lines.



Figure 2. Crust and skeleton of triangulation

(Gold, 1999) and (Gold and Snoeyink, 2001) simplified Amenta's algorithm for the extraction of the crust by showing that, in every Delaunay/Voronoi edge pair, either the Delaunay edge could be assigned to the crust or else the dual Voronoi edge could be assigned to the skeleton. The Delaunay edge belongs to crust when there exists a circle through its two vertices that does not contain either of its associated Voronoi vertices; if not then the corresponding Voronoi edge belongs to the skeleton. In practice their approach boils down to a simple InCircle test applied for each Delaunay/Voronoi edge pair, as shown in Fig. 3.

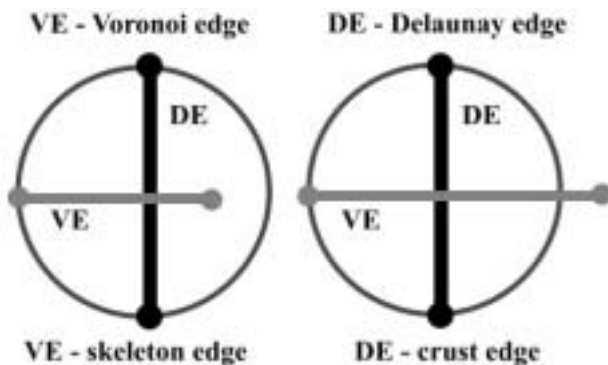


Figure 3. Results of crust/skeleton test

#### Generation of Ridge and Valley Lines

In our particular case the data is in the form of contour lines that we assume are sufficiently well sampled – perhaps derived from scanned maps. Despite modern satellite imaging, much of the world's data is still in this form. An additional property is not sufficiently appreciated – they are subjective, the result of human judgement at the time they were drawn. Thus they are clearly intended to convey information about the perceived form of the surface at a particular scale – and it would be desirable to preserve this, as derived ridges and valleys.

Fig. 4 shows our raw data set (which is completely imaginary), which we use to produce a Delaunay triangulation and a Voronoi diagram. Fig. 5 shows the crust and skeleton obtained from the triangulated data points. Because the sampling along contour lines is sufficient, the resulting crust reconstructs their

shape. Fig. 1c shows the crust and only those skeleton lines that provide unique information – ridge and valley lines that separate points on the same contour, rather than merely those points that separate adjacent contours. (Aumann *et al.*, 1991) produced somewhat similar results by raster processing.



Figure 4. Contours data points



Figure 5. Crust and skeleton of Fig. 4 data



Figure 6. Crust and skeleton branches of Fig. 4 data

Fig. 7a shows a close-up of the test data set, illustrating a key point of Amenta *et al.*'s work: if crust edges (forming the contour boundary) may not cross the skeleton, then inserting the skeleton points will break up non-crust triangle edges. In particular, if the skeletons between different contours are ignored, then insertion of the remaining branch skeleton points will eliminate all "flat triangles" formed from points of the same elevation. Thus ridge and valley lines are readily generated automatically. The same is true in the case of closed summits (Fig. 7b). The challenge is to assign meaningful elevation values to skeleton points.

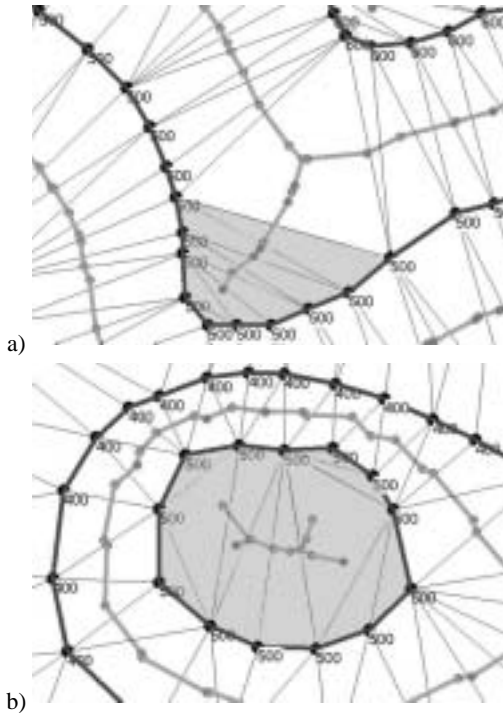


Figure 7. Skeleton and "flat triangles" – a) ridge; b) summit

Two techniques have been developed for this, each with its own physical interpretation. The first, following (Thibault and Gold, 2000), uses Blum's concept of height as a function of distance from the curve or polygon boundary, with the highest elevations forming the crest at the skeleton line (Blum, 1967). This is illustrated in Figs. 8a and 8b, where points on a simple closed curve are used to generate the crust and skeleton. In Fig. 8a, the circumcentres of the skeleton points are given a height above the previous contour level equal to the circumradius. The resulting interpolated model is shown in Fig. 8b. This model is based on the idea that all slopes are identical, and thus the radius is proportional to the height of the skeleton point. Of course, in the case of a real summit as in Fig. 7b, the slope would initially be unknown, and would be estimated using circumradii from the next contour level down (Thibault and Gold, 2000).

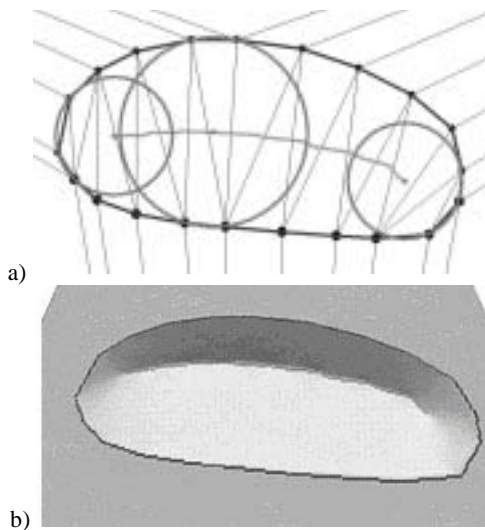


Figure 8. Triangulation of a summit – a) crust, skeleton and circumcentres; b) elevation model after adding skeleton vertices with assigned height values

In the case of a ridge or valley, the circumradius may also be used to estimate skeleton heights based on the hypothesis of equal slopes. In Fig 9, the larger circle, at the junction of the skeleton branches, has a known elevation – half way between the contours – and may be used to estimate the local slope. The elevation of the centre of the smaller circle is thus based on the ratio of the two radii. For details see (Thibault and Gold, 2000).

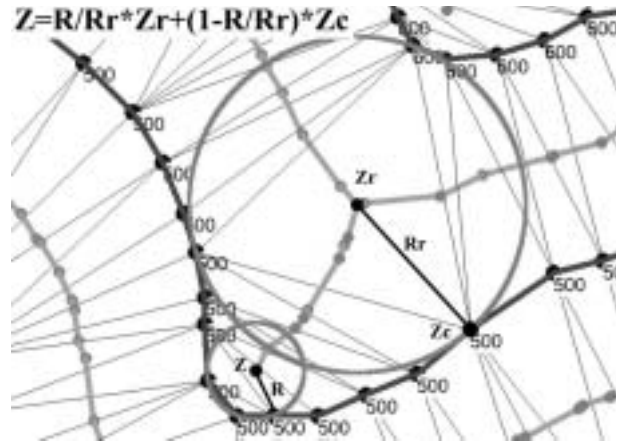


Figure 9. Estimating skeleton heights from circumradii

While this method is always available, it is not always the preferred solution where constant slope down the drainage valley, rather than constant valley-side slope, is more appropriate. In a second approach, illustrated in Fig. 10, the line of the valley is determined by searching along the skeleton, and heights are assigned based on their relative distance along this line. This may be complicated where there are several valley branches – in which case the longest branch is used as the reference line. This involves careful programming of the search routines, although the concept is simple. In practice, an automated procedure has been developed, which uses the valley length approach where possible, and the side-slope method when no valley head can be detected, such as at summits and passes.

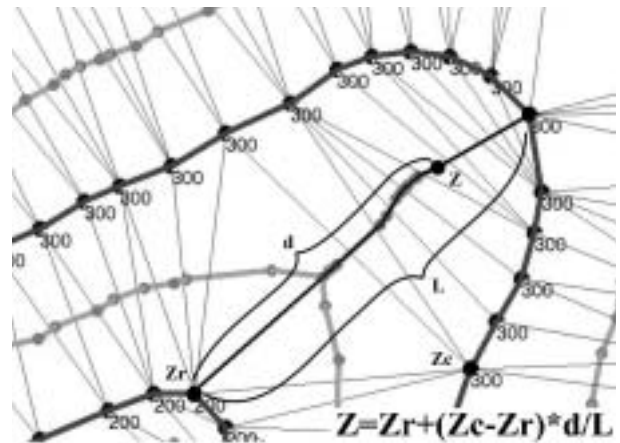


Figure 10. Estimating skeleton heights from valley length

### Components of an Interpolation Model

On the basis of a sufficient set of data points, we now wanted to generate a terrain model with satisfactory elevations and slopes,

as the basis of a valid rainfall runoff model. Our approach was to interpolate a height grid over the test area, and to view this with an appropriate terrain visualization tool. To obtain perspective views we used Genesis II, available from www.geomantics.com. Vertical views were generated using version 5 of the Manifold GIS, available from www.Manifold.net. We feel that 3D visualization has been under-utilized as a tool for testing terrain modelling algorithms, and the results are often more useful than a purely mathematical or statistical approach.

We have restricted ourselves to an evaluation of several weighted-average methods, as there are a variety of techniques in common that can be compared. All of the methods were programmed by ourselves – which left out the very popular Kriging approach, as too complicated, and not necessarily better. Nevertheless, many aspects of this study apply to this method as well, since it is a weighted-average method with the same problems of neighbour selection, etc., as the methods we attempted

In general, we may ask about three components of a weighted-average interpolation method. Firstly: what is the weighting process used? Secondly: what is the set of neighbours used to obtain the average? Thirdly: what is the elevation function being averaged? (Often it is the data point elevation alone, but sometimes it is a plane through the data point incorporating slope information as well.)

One simple weighted-average model is triangle-based interpolation in which we perform a linear interpolation within each triangle T and areas of three triangles formed inside triangle T by interpolated point and two triangle T vertices are used as weights, as shown in Fig. 11.

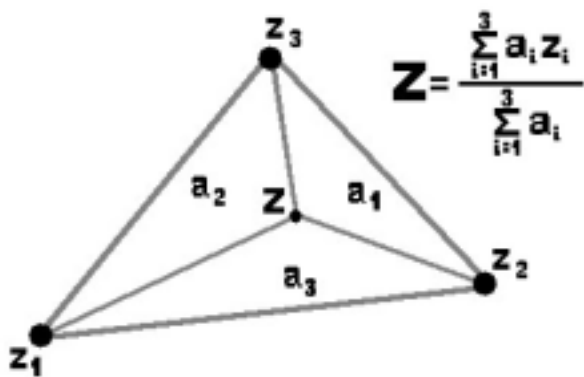
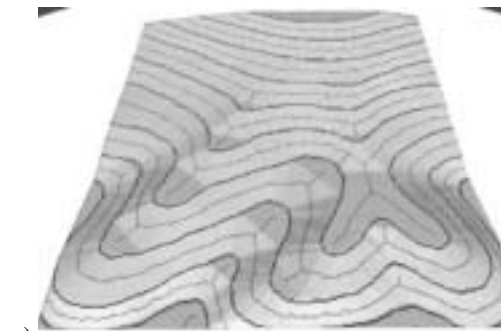
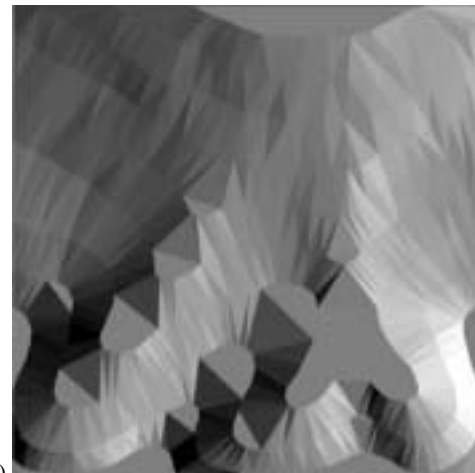


Figure 11. Triangle-based interpolation

Fig. 12a and 12b shows the result of applying this method for our data set. In perspective view in Fig. 12a the crust and skeleton are draped over the terrain. The flat triangles are readily seen in both pictures and it is easy to notice in Fig. 12a that skeleton branches are present in these flat areas. Fig. 13 shows the improved model when all flat triangles are removed by means of adding to triangulation estimated skeleton points. The triangle-based method is very fast but in the resulting surface there are sharp gradient breaks across triangles edges.

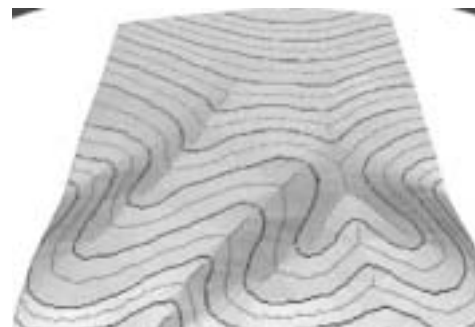


a)

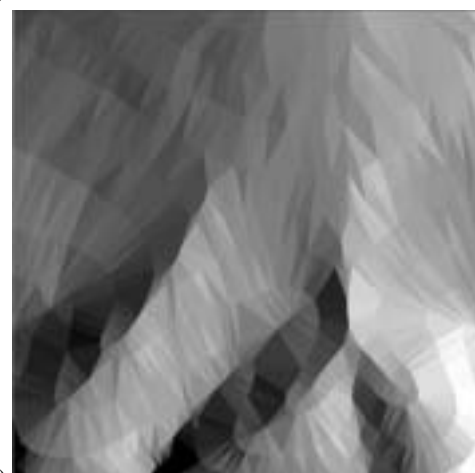


b)

Figure 12. Triangle-based interpolation from Delaunay triangulation – a) perspective view; b) vertical view



a)



b)

Figure 13. Triangle-based interpolation from Delaunay triangulation enriched with skeleton points – a) perspective view; b) vertical view

The other weighted average models that were tested were the traditional gravity model, and the more recent “area-stealing” or “natural neighbour” or perhaps more properly “Sibson” interpolation methods (Gold 1989; Sibson, 1980; Watson and Philip, 1987).

In the case of the gravity model the weighting of each data point used is inversely proportional to the square of the distance from the data point to the grid node being estimated, although other exponents have been used. There is no obvious set of data points to use, so one of a variety of forms of “counting circle” is used. Fig. 14 gives an idea of this method for an interpolated point having four neighbours selected by a circle of specified radius.

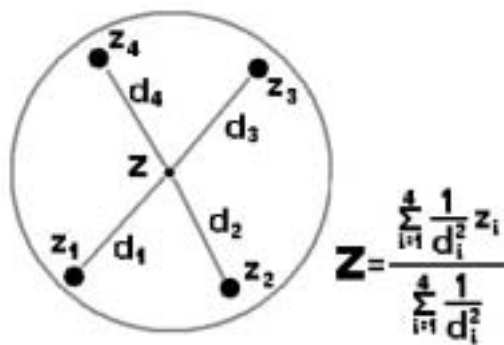


Figure 14. Gravity interpolation

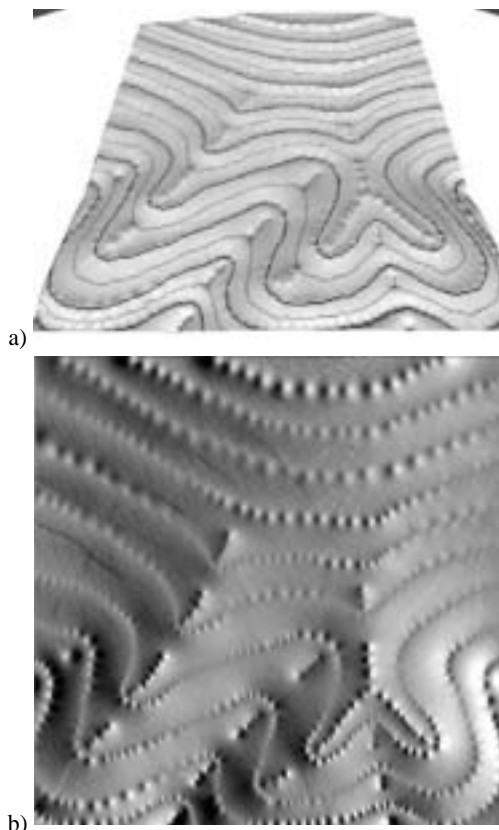


Figure 15. Interpolation using the gravity model – a) perspective view; b) vertical view

Fig. 15 shows the resulting surface for a radius of about a quarter of the map. Data points form bumps or hollows. If the radius is reduced there may be holes in the surface where no

data is found within the circle. If the radius is increased the surface becomes somewhat flattened, but the bumps remain. The result depends on the radius, and other selection properties, being used. Clearly, in addition, estimates of slope would be very poor, and very variable.

The Sibson method is based on the idea of inserting each grid point temporarily into the Voronoi diagram of the data points, and measuring the area stolen from each of a well-defined set of neighbours. These stolen areas are the weights used for the weighted-average. In our case we do not modify the triangulation during this interpolation process. In order to obtain stolen areas we only simulate the insertion, instead of temporarily inserting and deleting the interpolated point. Fig. 16 shows this process for a sample data set – the Z value is estimated using areas stolen from five neighbouring Voronoi cells.

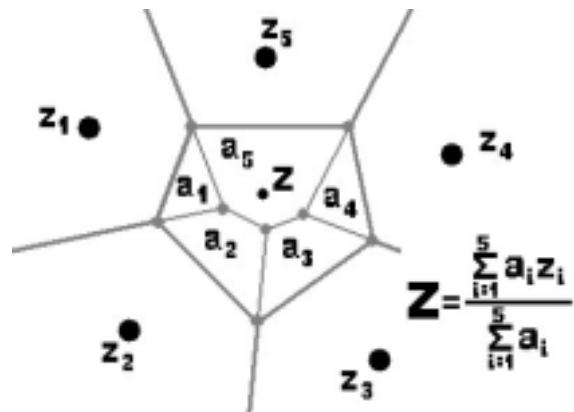


Figure 16. Sibson interpolation

Fig. 17 shows a sample data set and the neighbour selection for the same point in both the gravity and Sibson method. In the Sibson method natural neighbour selection results in a reasonable set of neighbours, but the circle used in the gravity method may not select a sufficient number of neighbours to produce a valid elevation value for the interpolated point.

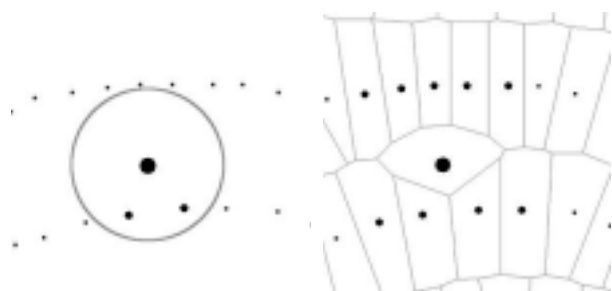


Figure 17. Neighbour selection a) using a counting circle; b) using Voronoi neighbours

The Sibson method is particularly appropriate for poor data distributions as the number of neighbours used is well defined. In the gravity model, when the data distribution is highly anisotropic, there is considerable difficulty in finding a valid counting circle radius.

Fig. 18 shows the results of using Sibson interpolation. The surface behaves well - it fits the original data and is smooth in areas between data points, but is angular at ridges and valleys. Indeed, slopes are discontinuous at all data points (Sibson, 1980).

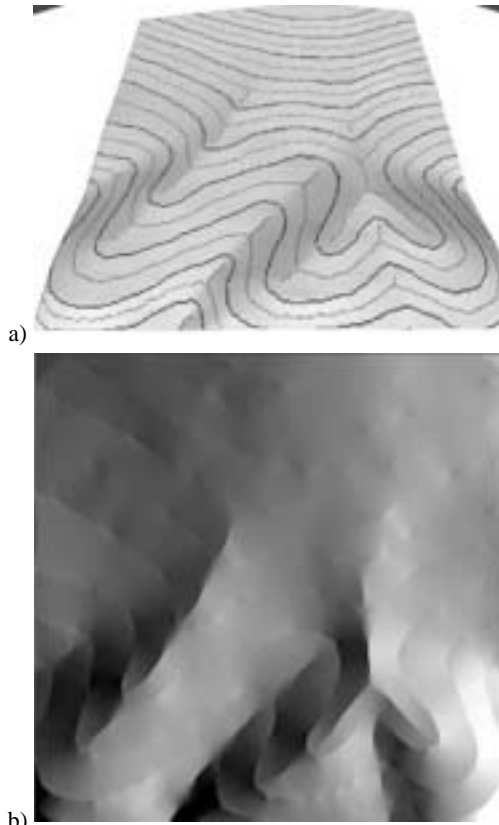


Figure 18. Sibson interpolation – a) perspective view; b) vertical view

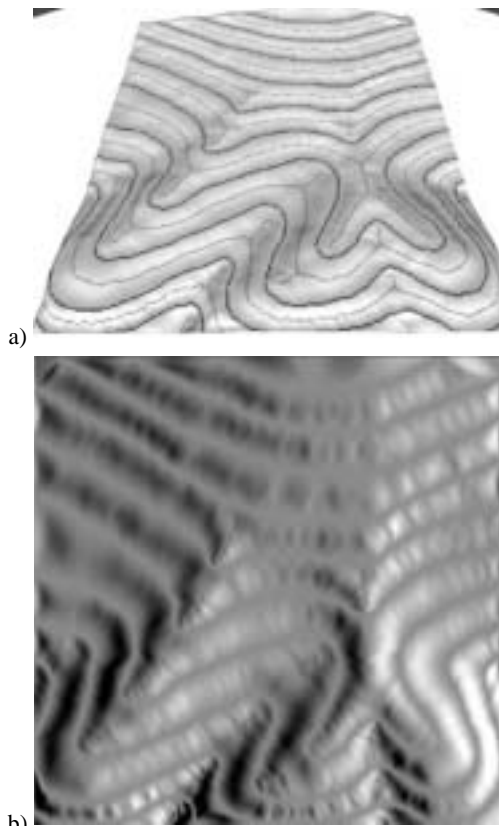


Figure 19. Adding smoothing to Sibson interpolation – a) perspective view; b) vertical view

To remove discontinuities at data points, one solution is to re-weight the weights, so that the contribution of any one data point not only becomes zero as the grid point approaches it, but the slope of the weighting function approaches zero also (Gold, 1989). In our case we replace every weight  $w$  by a new weight  $w'$  obtained by means of a simple function:

$$w' = 3 * w^2 - 2 * w^3$$

Fig. 19 shows the effect of adding this smoothing function. While the surface is smooth, the surface contains undesirable “waves” – indeed, applying this function gives a surface with zero slope at each data point.

### Slopes – The Ignored Factor

This brings us to a subject often ignored in selecting a method for terrain modelling – the slope of the generated surface. In real applications, however, accuracy of slope is often more important than accuracy of elevation – for example in runoff modelling and erosion. Clearly an assumption of zero slope at each data point, as above, is inappropriate. However, in our weighted-average operation we can replace the height of a neighbouring data point by the value of a function defined at that data point – probably a planar function involving the data point height and local slopes. The idea of this process is shown in Fig. 20 - point P at location  $(x,y)$  is being estimated and  $P_i$  is one of its neighbours. During interpolation the  $z$  value of each neighbouring data point  $P_i$  is replaced by value  $z'$  of the plane (tangent to the surface at  $P_i$ ) at location  $(x,y)$ . Thus at any grid node location we find the neighbouring points and evaluate their planar functions for the  $(x, y)$  of the grid node. These  $z$  estimates are then weighted and averaged as before.

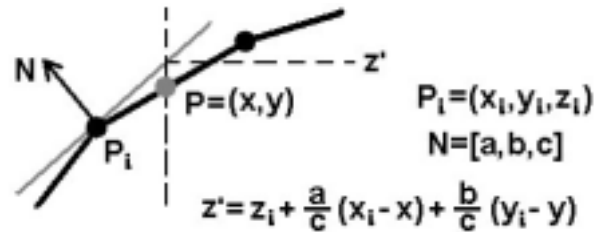


Figure 20. Plane value at location of interpolated point P

Fig. 21 shows the result of using Sibson interpolation with data point slopes. The form is good, but slight breaks in slope can be seen at contour lines. When using smoothing and slope information together, the surface is smooth, but has unwanted oscillations, see Fig. 22. Clearly an improved smoothing function is desirable to eliminate these side-effects.

When the sample points are closely spaced along the contours adding slopes to the simple TIN model, as in Fig. 23a, produces results that are almost as good as the Sibson method. However, the Sibson method is much superior for sparser data, or where the points do not form contour lines. The gravity model does not provide particularly good slope estimates, but even here including the data point slope function produces a significant improvement, as in Fig 23b.

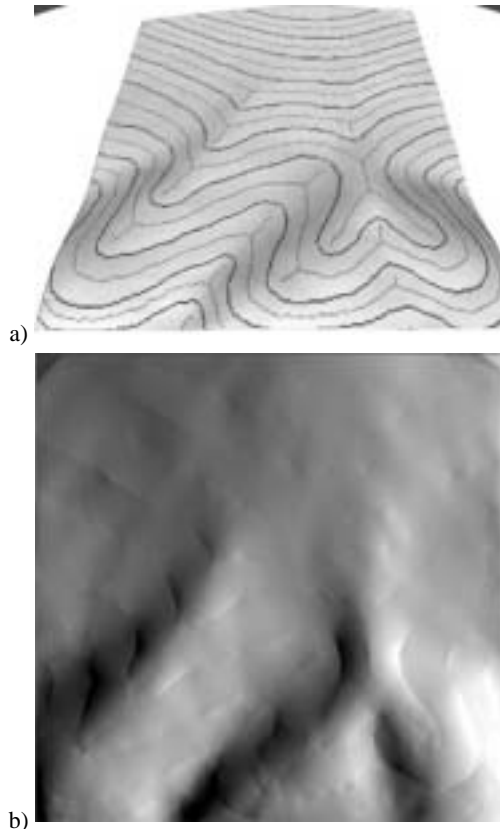


Figure 21. Sibson interpolation using slopes at data points – a) perspective view; b) vertical view

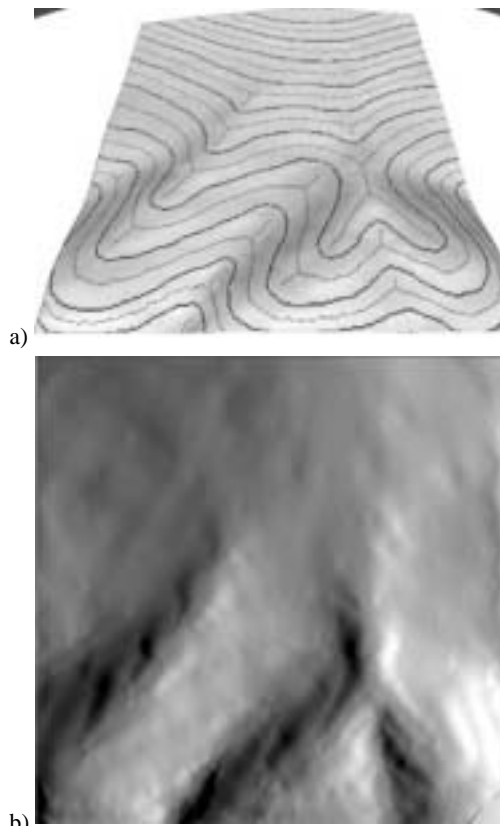


Figure 22. Sibson interpolation using slopes and smoothing function at data points – a) perspective view; b) vertical view

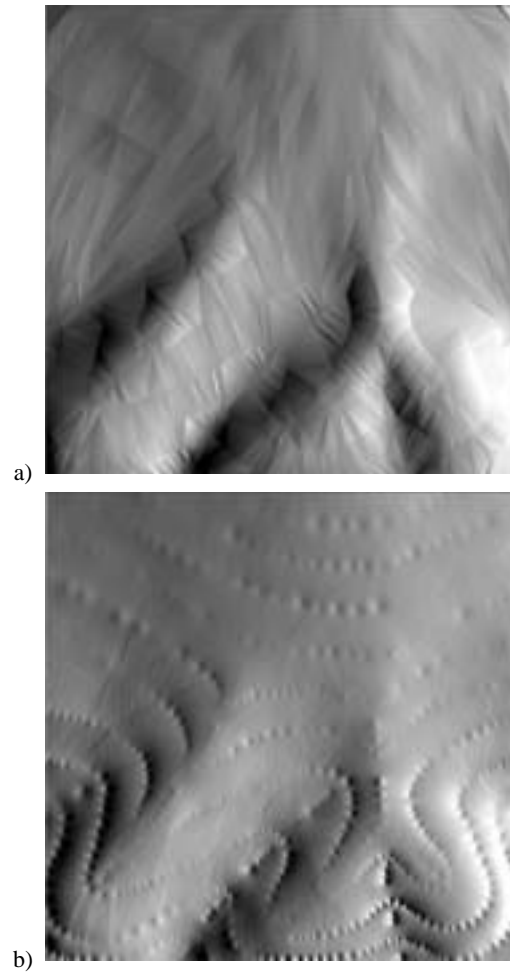


Figure 23. Adding slopes at data points - a) triangle-based interpolation; b) gravity interpolation

### Proposed methodology and conclusions

For the common problem of deriving surfaces from contours, we propose a general approach:

1. Generate skeleton points along the ridges, valleys, pits, summits and passes by the method of (Aumann *et al.*, 1991) or of (Thibault and Gold, 2000).
2. Assign elevations to these skeleton points by the methods described here, or other suitable techniques.
3. Eliminate flat triangles by the insertion of these skeleton points into the original TIN.
4. Estimate slope information at each data point by any appropriate technique.
5. Perform weighted-average interpolation using the previously estimated slope information. Avoid methods, such as the gravity model, with exponentially large close-range weightings, and avoid neighbour selection techniques which require user-specified parameters, such as counting circle radius.

Surprisingly, mathematically guaranteed slope continuity is not usually critical, although we are continuing to work on an improved smoothing function that guarantees both slope continuity and minimum curvature – probably based on the work of (Anton *et al.*, 1998). Nevertheless, the moral is clear: both for finding adjacent points and for skeleton extraction, a

consistent definition of neighbourhood is essential for effective algorithm development.

We conclude with another imaginary example. Fig. 24a shows the surface obtained from a small set of contours by the triangle-based interpolation. Instead of the expected four small hills defined by their contours, flat areas are produced on summits of the hills and in valleys between them. Fig. 24b shows the result using Sibson interpolation with slopes at data points and skeletons. Skeleton heights were obtained using circumscribed circles, as no valley-heads were detected. While our evaluation was deliberately subjective, we consider that our results in this case, as with the previous imaginary landform, closely follow the perceptual model of the original interpretation. Thus, for the reconstruction of surfaces from contours, we believe that our methods are a significant improvement on previous work.

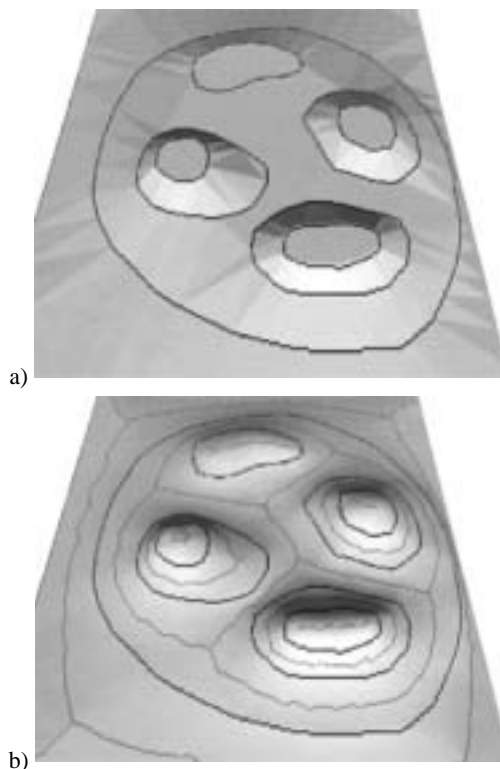


Figure 24. Triangulation of several small hills - a) triangle based interpolation; b) Sibson interpolation with slopes

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