Improving 9-Intersection Model by Replacing the Complement with Voronoi Region

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Abstract

9-intersection model is the most popular framework used for formalizing the spatial relations between two spatial objects A and B. It transforms the topological relationships between two simple spatial objects A and B into point-set topology problem in terms of the intersections of A’s boundary (symbol 182 \f "Symbol" \s 11∂}A), interior (A^0) and exterior (A^-) with B’s boundary (symbol 182 \f "Symbol" \s 11∂}B), interior (B^0) and exterior (B^-). It was shown in the this paper that there exist some limitations of the original 9-intersection model due to its definition of an object’s exterior as its complement, including being difficult to distinguish different disjoint relations and relations between complex objects with holes, difficult or even impossible to compute the intersections with the two object’s complementssymbol 182 \f "Symbol" \s 11∂}AB^-, A^0B^-, A^-symbol 182 \f "Symbol" \s 11∂}BA^-B^0 and A^-B^-) since the complements are infinitive. The authors suggested to re-define the exterior of spatial object by replacing the complement with its Voronoi region. A new Voronoi-based 9-intersection (VNI) was proposed and used for formalizing topological relations between spatial objects. By improving the 9-interscetion model, it is now possible to distinguish disjoint relations and to deal with objects with holes. It is also possible to compute the exterior-based intersections and manipulate spatial relations with the VNI.

1. Overview of the original 9-intersection model

The spatial relations between spatial entities are known as important as the entities themselves. It is therefore very essential to know *what* possible spatial relationships are and *how* they can be determined. The 9-intersection model is the most popular mathematical framework for formalizing spatial relations and have been used for a number of applications in spatial query languages [Engenhofer,1991, Clementini *et al*. 1994, Mark *et al*. 1995]. It transforms the topological relationships between two simple spatial objects A and B into point-set topology problem. That is, the topological relations between two objects A and B are defined in terms of the intersections of A's boundary (symbol 182 \f "Symbol" \s 11∂}A), interior (A^0) and exterior (A^-) with B's boundary (symbol 182 \f "Symbol" \s 11∂}B), interior (B^0) and exterior (B^-), as shown in Fig.1. By considering whether the contents of the 9-intersection is empty or non-empty, a number of binary topological relations could be identified [Egenhofer and Franzosa, 1991]. For instance, eight relations can be realized between two spatial regions in R^2 , such as *disjoint*, *meet*, *equal*, *inside*, *contains*, *covers*, *coverdBy* and *overlap*[Egenhofer and Sharma,1993]. It provides a complete coverage and the relations are mutually exclusive, that is, only one of them holds for any particular configurations at a time. Efforts had been made to examine the actual number of realizable topological relations between area-line, line-line, area-point, line-point, as well point-point [Egenhofer and Herring, 1991; Egenhofer, 1993; Sun et. al, 1993].

Fig.1 Point-set topology-based 9-intersection Model

It was further found that the number of relations existing among objects depends on the dimension of the space with respect to the dimension of t he objects and on topological properties of the objects embedded in that space [Egenhofer and Sharma,1993]. A dimension extended method was proposed by Clementini et. al. to take into account the dimension of the result of the intersection instead of only distinguishing empty or non-empty intersections [Clementini et.al., 1993]. The dimension extended method was also used in formalizing topological relations in 3d space [Guo and Chen, 1997]. On the other hand, the above model deals with simple objects as the homogeneously 2-dimensional, connected areas and lines with exactly two end points [Egenhofer,1993?]. In order to identify the topological relations between two regions with holes, each of these regions should be separated into its generalized region and its holes, and the combinatorial intersections of the generalized regions and holes would be examined [Egenhofer et. al. 1994]. Moreover, the 9-intersection model has also been used or extended for examining the possible topological relations between regions in discrete space [Egenhofer and Sharma,1993; Winter, 1995], modeling conceptual neighborhoods of topological line-region relations [Egenhofer and Mark, 1995], grouping the very large number of different topological relationships for point, line and area features into a small sets of meaningful relations [Clementini et.al., 1993], describing the directional relationships between arbitrary shapes and flow direction relationships [Abdelmoty and Williams, 1994; Papadias and Theodoridis, 1997], deriving the composition of two binary topological relations [Egenhofer,1991], describing changes to topological relationships by introducing a Closest-Topological-Relationship-Graph and the concept of a topological distance [Egenhofer and Al-Taha, 1992], analyzing the distribution of topological relations in geographic datasets [Florence III and Egenhofer, 1996], as well as formalizing the spatio-temporal relations between the father-son parcels during the process of land subdivision [Chang and Chen, 1997]. The findings of these investigations have significantly contributed to the development of the state-of-art spatial data models and spatial query functionalities [Egenhofer and Mark, 1995; Mark et. al., 1995; Papadis and Theodoridis, 1997].

It is known that the 9-intersection model is an extension of the initially proposed 4-intersection model which has sets of intersections symbol 182 \f "Symbol" \s 11∂}Asymbol 182 \f "Symbol" \s 11∂}B, symbol 182 \f "Symbol" \s 11∂}AB^0 , symbol 182 \f "Symbol" \s 11∂}BA^0 and A^0B^0. By adding the intersections with the two object's complements, the 9-intersection could distinguish among topological relations that would be considered the same using the 4-intersection when two objects have co-dimension 0 [Egenhofer et.al., 199?]. 33 different spatial relations are possible for two simple lines and 19 are possible for a line and a region. However, it would be shown in the second section of this paper that there exist some limitations due to the definition of an object's exterior as its complement. One is that it is difficult to distinguish different disjoint relations and relations between complex objects with holes. In addition, it is quite difficult or impossible to compute the intersections with the two object's complementssymbol 182 \f "Symbol" \s 11∂}AB^-, A^0B^-, A^-symbol 182 \f "Symbol" \s 11∂}BA^-B^0 and A^-B^-) since the complements are infinitive. The consequence is that although compound spatial relations could be defined in the SDTS with aggregates of two or more 9-intersection primitives [Mark et. al., 1995], but it is difficult to calculate these primitives from spatial data. By re-defining the exterior of spatial object with its Voronoi region instead of its complement, a new Voronoi-based 9-intersection (VNI) was proposed in the third section. With the new VNI, it is possible to distinguish disjoint relations, to deal with objects with holes and to compute the five exterior-based intersections. Further investigations are also discussed in the 5th section of this paper.

2. Revisiting roles of complement in distinguishing spatial relations

The complement of an object is the set of all points of R^2 not contained in that object [Egenhofer, 1991]. There are five complement-related intersections in the 9-intersection model, symbol 182 \f "Symbol" \s 11∂}AB^-, A^0B^-, A^-symbol 182 \f "Symbol" \s 11∂}B, A^-B^0, and A^-B^- . It was pointed out that the complement is useful for judging whether or not an object is completely included in another one if the co-dimension is greater than 0 [Egenhofer et.al., 199?]. In Fig.2, we have two groups of line-line relations. The relations in each group have the same 4-intersection, but distinct 9-intersection. In order to revisit the roles of the complements A^- and B^- ,we'll examine how the complements behave in distinguishing spatial relations for the area-area, line-line and line-area objects.

[Fig. 2 From 4-intersection R_4(A,B) to 9-intersection R_9(A,B)]

It was shown in Fig.3a that all the five complement-related sets remain non-empty when the relations between two area objects change gradually from disjoint, meet to partially overlap. When one area object falls into the another one (such as equal, covered-by and cover, contains and inside), some of the five complement-related sets would be empty. In fact, A's complement will not interact with the B's boundary and interior when A and B are equal, and vice versa. When A is covered by object B, its boundary and interior fall into B's interior, and will not intersect with B's complement. It should be pointed out that the eight relations in Fig.3a have distinct 4- intersections. It would say that the complements do not have critical roles in distinguishing region-region relations, and that's why the 9-intersection reveals the same set of region-region relations.

The 9-interscetion results in 33 distinct line-line relations in IR^2 and provides a much finer resolution than the 4-intersection[Egenhofer and Herring, 1991]. When the two simple line objects have disjoint, meet, cross and overlap relations as shown in Fig.3b, all the five complement-related sets remain non-empty. When a line falls into another one, some of the complement-related sets would be empty. For instance, when A and B have a cover-relation, A's boundary(only two nodes) and interior are contained by B's interior and they do not intersect with the complement of B. A^-symbol 182 \f "Symbol" \s 11∂}B and A^-B^0 are therefore empty. It is interested to see that the overlap and cover relations have the same 4-intesection, but have distinct 9-line intersection. Moreover, it can also be seen from Fig. 3c that five complement-related sets play roles when a line's interior is falling into the other's.

[Fig. 3a Roles of complements in distinguishing area objects]

[Fig. 3b Roles of complements in distinguishing line-objects]

[Fig. 3c Roles of complements in distinguishing line-area objects]

It is interested to note that $A^{-}B^{-}$ always take the non-empty value -symbol 198 \f "Symbol" \s 11∅}. The reason is that the complements of any two objects are largely overlapped. One of the consequences is that it is difficult to distinguish disjoint relations. For instance, there are four objects A, B, C and D in Fig.4. These objects have different disjoint relations, but the 9-intersections R_{AB}, R_{AC} and R_{AD} are same. Due to the infinitive complement, A's complement intersects with the boundaries, interiors and complements of object B, C and D, and it's boundary and interior also intersect with the complements of objects B, C and D. The five complement-related sets always take the non-empty value -symbol 198 \f "Symbol" \s 11∅}. In other words, the complements could not play roles in distinguishing these disjoint relations. Another example is given in Fig.5 where three different disjoint relations between object A and object B have the same 9-interscetion. It seems that the original point-set topology based 9-intersection is good at distinguishing the intersected spatial relations where objects intersect. All the disjoint cases will be mapped into the same 9-inersection due to the definition of the exterior of an object as its complement. Since about 80% spatial relations are disjoint relations [Florence and Egenhofer, 1996], it is necessary to improve the original 9-intersection model to distinguish and describe these disjoint relations.

[Fig. 4 The complements of any two objects are largely overlapped]

[Fig. 5 Different disjoint relations]

The second consequence is that it is difficult or even impossible to calculate the five complement-related intersections symbol 182 \f "Symbol" \s 11∂}AB^{-}, $A^{0}B^{-}$, A^{-}symbol 182 \f "Symbol" \s 11∂}B , $A^{-}B^{0}$ and $A^{-}B^{-}$ since the complement of an object is the set of all points of R^2 not contained in that object. On one hand, it is difficult to calculate the intersections of two object's components from their geometric data automatically. Qualitative analysis is required for deriving the 9-intersections. On the other hand, it is also difficult to derive automatically those spatial objects who satisfying given 9-intersection. In other words, it is difficult to manipulate the spatial relations with the 9-intersections.

The 9-interscetion model has also limitations in distinguishing relations between complex objects with holes. For the cover and cover-by relations between A and B in Fig. 6a, two distinct 9-intesections can be found. In that case, A and B are simple homogeneously 2-dimensional, connected areas objects [Egenhofer and Franzosa 1991, Clementini et al. 1993]. However, the 9-intersection would be the same when A or B has a hole, as shown in Fig.6b. The interior and boundary of the object (i.e., A) falling in the hole do not intersect the interior of the other object (i.e., B), so symbol 182 \f "Symbol" \s 11∂}AB^0=symbol 198 \f "Symbol" \s 11∅}, A^0B^0 =symbol 198 \f "Symbol" \s 11∅} , and A^0symbol 182 \f "Symbol" \s 11∂}B =symbol 198 \f "Symbol" \s 11∅} . Moreover, the five complement-related intersections symbol 182 \f "Symbol" \s 11∂}AB^-, A^0B^-, A^-symbol 182 \f "Symbol" \s 11∂}B , A^-B^0 and A^-B^- all take non-empty values. Some other examples are illustrated in Fig.6c, 6d and 6e.

[Fig.6 Problems caused when the area object with holes]

3. Redefining the exteriors by replacing A^- with A^v

We propose here to redefine the exterior of an object with the Voronoi region of that object. The whole space is subdivided into a set of Voronoi regions (or tiles) according to the distribution of the objects in the Voronoi diagram, which is one of the most fundamental data structures in computational geometry and is a space-filling topological structure [Aurenhammer 1991]. Each spatial object has its own ‘influence region’ (Voronoi region) containing all locations closer to that object than to any other, as shown in Fig.7. It was found that Voronoi diagram is another spatial model which poses a variety of challenges to the ‘usual way of doing things’ in GISs [Gold, 1989, 1991,1992; Wright and Goodchild, 1997]. The application of Voronoi diagram in GIS spatial data modeling and analysis has been investigated during the past few years [Yang and Gold, 1995; Gold et.al., 1996; Edwards et.al., 1996; Hu and Chen, 1996; Chen and Cui, 1997].

[Fig.7 Voronoi diagram of point, line and area objects]

Suppose we have a set of spatial objects, SO={O_1,…, O_i ,…, O_n} {1 symbol 163 \f "Symbol" \s 11≤} n symbol 163 \f "Symbol" \s 11≤} symbol 165 \f "Symbol" \s 11∞} }, in IR^2, O_i may be a point object O_P, or line object O_L or area object O_A. An area object is not necessarily convex, and may have holes in which another area may exist. The object Voronoi region of O_i (called O^v) can be defined as

V(O_i)={psymbol 124 \f "Symbol" \s 11|ds(p, O_i) symbol 163 \f "Symbol" \s 11≤} ds(p,O_j),

jsymbol 185 \f "Symbol" \s 11≠}i, i, jsymbol 206 \f "Symbol" \s 11∈} In }

From the properties of Voronoi diagram, these Voronoi regions are finite if a bounded region S is defined. In addition, each Voronoi region has limited adjacent regions. By replacing the complement of an object with its Voronoi region, it is possible to overcome the limitations of the original 9-interscetion model. And we have now a new Voronoi-based Nine-Intersection (called VNI briefly) framework as the following:

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^o & \partial A \cap B^V \\ A^o \cap \partial B & A^o \cap B^o & A^o \cap B^V \\ A^V \cap \partial B & A^V \cap B^o & A^V \cap B^V \end{bmatrix} (1)$$

where A^v is object A's Voronoi region, B^v is object B's Voronoi region. symbol 182 \f "Symbol" \s 11∂}AB^v, A^0B^v, A^vsymbol 182 \f "Symbol" \s 11∂}B, A^vB^0 and A^vB^v are the five new intersections related to Voronoi regions.

A^vB^v would be non-empty when two objects are adjacent, such as A and B in Fig. 8a. It is because the Voronoi region of A share the same boundary with that of B. When there is an object C between A and B (Fig.8b), their Voronoi regions are separated by that of C and A^vB^v is empty. It is therefore possible to distinguish the adjacent relations from other disjoint relations with the VNI. More adjacent relations could be defined and derived with the Voronoi diagram, such as immediate neighbor, nearest neighbor, second-nearest, lateral neighbor, tracing neighbor, etc. [Chen et. al., 1997].

[Fig.8 Distinguishing disjoint relations with VNI]

When the boundary of object A meets with that of object B, their Voronoi regions would also meet according to the definition of Voronoi diagram. symbol 182 \f "Symbol" \s 11∂}Asymbol 182 \f "Symbol" \s 11∂}*B* and A^vB^v would both take non-empty values in this case as shown in Fig.9a. In addition, the boundary of object A meets with B^v and B's boundary meet with A^v. The Voronoi-based 9-intersection for the *meet* relation between area objects is therefore different from the original 9-intersection. If object A's boundary meets the inner boundary of B which has a hole as shown in Fig.9b, A's Voronoi region intersects B's inner boundary resulting in symbol 182 \f "Symbol" \s 11∂}Asymbol 182 \f "Symbol" \s 11∂}*B*=-symbol 198 \f "Symbol" \s 11∅} , A^vsymbol 182 \f "Symbol" \s 11∂}*B* =-symbol 198 \f "Symbol" \s 11∅}, A^vB^v =-symbol 198 \f

"Symbol" \s 11∅} and symbol 182 \f "Symbol" \s 11∂}AB^v =-symbol 198 \f "Symbol" \s 11∅}. Moreover, the whole body of A is contained in the hole of B, we say A's interior overlaps with the convex of B and A^0B^v =-symbol 198 \f "Symbol" \s 11∅}. The example shown in Fig.9b has the same original 9-intersection, but has a different Voronoi-based 9-intersection than Fig.9a. The example illustrated in Fig.9d is a *contained-by* relation which has the same 9-intersection with the *contains* relation shown in Fig.9e. The Voronoi regions touch and there is not intersection of boundaries and interiors between the two objects. However, the boundary and interior of the contained object intersect with the Voronoi convex of the other object. Another example is given by Fig. 9f where a line meets a homogeneously 2-dimensional and connected area B and the line falls into an area's hole in Fig.9g. It can be seen from these examples that it is possible to distinguish relations between complex objects with holes using the Voronoi-based 9-interscection model.

[Fig.9 Distinguishing relations with VNI]

Both vector and raster approaches have been developed for generating Voronoi diagram of points, lines and areas [Gold and Yang, 1995, Li, et.al., 1998]. Area Voronoi diagram is constructed from its corresponding line Voronoi diagram in the vector-based Voronoi diagram [Okabe et.al., 1992, pp.183; Gold and Yang, 1995]. It is generated directly by defining "distance between objects" in the raster Voronoi diagram [Li, et.al., 1998]. With these generating approaches, it is possible to calculate the Voronoi region of a given spatial object (point, line or area). The Voronoi boundary and interior of an area object could also be derived from the Voronoi diagram. It is therefore feasible to calculate the five complement-related intersections symbol 182 \f "Symbol" \s 11∂}AB^-, A^0B^-, A^-symbol 182 \f "Symbol" \s 11∂}B , A^-B^0 and A^-B^- automatically form spatial data.

4. Formalizing topological relations with VNI

Topological relations between point, line and area objects have been formalized with the new VNI model, including relations between area-area, line-line, line-area, point-point, point-line and point-area objects. Some of the results are listed in Table 1. Among the thirteen topologically distinct relationships between two areas characterized with the VNI, seven of them could not be distinguished using the original 9-interscetion model. Some of the distinguished relations between line-line objects are shown in Fig.10.

Table 1 Distinguished topological relationships using VNI

	Case	Result
AA	Area/Area	13
LL	Line/Line	8
LA	Line/Area	13
PP	Point/Point	3
PL	Point/line	4
PA	Point/Area	5

[Fig.10 Part of relations between line-line objects with VNI]

Abdelmoty et.al. classified the approaches for formalizing spatial relations into two categories. [Abdelmoty and Williams, 1994]. One is the *intersection*-based model where an object is represented in terms of its components, and relationships are the result of the combinatorial intersection of those components. The original 9-intersection model is such a model that spatial relations between objects considered are the result of the exhaustive combinatorial intersection of their components. The other is the *interaction*-based approach, where the body of an object is considered as a whole and is not decomposed into its components. It is interesting to note that the Voronoi-based 9-intersection model proposed here has integrated the *intersection* and *interaction* approaches. The four sets symbol 182 \f "Symbol" \s 11∂}Asymbol 182 \f "Symbol" \s 11∂}B, symbol 182 \f "Symbol" \s 11∂}AB^0 , symbol 182 \f "Symbol" \s 11∂}BA^0 and A^0B^0 of the VNI can take the intersection between objects into account. By generating Voronoi region for the whole body of each spatial object, the interactions between adjacent objects can be distinguished with the five sets symbol 182 \f "Symbol" \s 11∂}AB^v, A^0B^v, A^vsymbol 182 \f "Symbol" \s 11∂}B, A^vB^0 and A^vB^v.

5 Further investigations

From the above discussions, it is clear that the 9-interscetion model can be improved if the exterior of an object would be redefined by replacing its complement with its Voronoi region. One of the advantages of the new Voronoi-based 9-interscetion is that it is possible to distinguish disjoint relations since each object has limited neighbors instead of having relations with all other objects. Being able to deal with objects with holes is another advantage of the VNI. Moreover, it is possible or easier to compute the five exterior-based intersections (i.e., symbol 182 \f "Symbol" \s 11∂}AB^v, A^0B^v, A^vsymbol 182 \f "Symbol" \s 11∂}B, A^vB^0 and A^vB^v) since the Voronoi regions of each object can be generated and manipulated. It makes it easier to query and manipulate about the topological relations between two given objects with the new 9-interscetion model. However, lots of issues regarding the proposed VNI model remain to be examined and developed.

A. Comparison of the results of VNI with the original 9-intersection

Detailed analysis of the relations distinguished by the VNI and comparison with the results realized by the original 9-interscetion model is under investigation. While the original 9-intersection model deals with the simple objects defined as homogeneously 2-dimensional, connected areas and lines with exactly two end points, the area objects used in the VNI may have holes. The definition needs to be extended for covering the kinds of geometric objects, such as loop objects. Moreover, the conceptual neighborhoods of topological relations, Closest-Topological-Relationship-Graph for describing changes to topological relationships, directional relationships between arbitrary shapes and flow direction relationships will be examined with the VNI.

B. Development of a special toolkit for manipulating spatial relations with VNI

It is very helpful if the 9-intersection and its corresponding semantics of spatial relations could be derived when two objects are located. On the other hand, when a 9-intersection is given, it is also very essential to retrieve the spatial objects who satisfy such spatial relation. A special toolkit is under development by the authors now for derive the VNI from the geometry or spatial location of geographic entities. With this tollkit, users can then examine whether a specific spatial relation occurs with their expectations and GIS researchers can easily investigate what spatial relation exist for a given spatial dataset.

C. Inferencing spatial relations with the VNI

Since the Voronoi diagram forms a pattern of packed convex polygons covering the whole space, all spatial objects are linked together by their Voronoi regions. This makes it possible to deduce or reason about the relations between any two spatial objects with the Voronoi tessellation. The constraints among objects under a given spatial relation can be translated into VNI templates (or primitives), i.e, a pattern of empty, non-empty or arbitrary intersections representing the constraints among interiors, boundaries and Voronoi regions. Aggregates of two or more 9 VNI templates would be used for defining the compound spatial relations. One of the applications would be deriving the contiguity, connectivity and inclusion directly from the spaggette data set.

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