Title: **A Surface-Representation Approach to a Three-Dimensional Cadastre.**

First Paragraph

Cadastral systems are traditionally two-dimensional. However, there is now much talk of extending them into the third dimension, in part by incorporating the elevation information in a terrain model. Terrain models have been around for quite some time - and so have the arguments between using a grid (or DEM-Digital Elevation Model, in American parlance), or a triangulation (TIN-Triangulated Irregular Network). Usually either approach may be satisfactory-but there are increasing complaints that it is too restrictive, allowing only one z value for each (x, y) location, thus prohibiting the representation of caves, bridges, etc. Nevertheless, computer games everywhere bear witness to the fact that we can represent the most complex monsters-so what are we missing?

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Body Text

The solution, we believe, comes from examining the basis of one type of CAD-Computer Aided Design system-the kind that maintains a connected surface representation (boundary representation, or ‘b-rep’) of the object being modelled. Surprisingly, we found a great similarity between our well-known TIN model and the basic b-rep operations (known as Euler Operators, as they preserve the Euler-Poincaré formula relating vertices, edges, faces, holes, etc. for closed surfaces). This allows us to modify our terrain surfaces as never before!

Our first step was to examine these CAD operations in detail. Two points arose. Firstly, they were primarily intended for designing machine parts or things of that nature, and allowed for the creation of holes within individual flat faces. In our case, we were satisfied with constructing our surfaces from triangles, so some of this was unnecessary. Secondly, the underlying data structures usually used were very complex (the ‘winged-edge’ structure-see Baumgart (1972), Mantyla(1988)). We dealt with this by going to the Computing Science literature, and implementing the ‘Quad-Edge’ structure of Guibas and Stolfi (1985). This is frequently used for triangular mesh construction (including terrain models), and has a variety of extremely elegant (and simple) features that will be described later.

The result of this re-think is a three-layer design that incorporates the TIN model, but permits its extension to the more complex forms that are necessary for the integration of man-made features into the terrain. The base layer is the Quad-Edge structure with its two fundamental operations (Make-Edge and Splice) that permit the maintenance of any connected graph on a surface. The middle layer uses these operations to define three Euler Operators that suffice to maintain a simple b-rep surface model (without holes). The top layer uses these Euler Operators to build the standard TIN model (again with three commands).

So far this is not very exciting. The value of this analysis comes when we wish to construct additional features. Simple box-like buildings can be constructed using the available Euler Operators and, with the addition of one additional Operator, bridges and tunnels may easily be constructed. At the present stage of our work these are constructed interactively, but we hope later to automate some of this process.

It should be noted that we are not merely providing a visualization of ‘terrain with trees and buildings’, for example, where each object was composed separately and then artfully placed so that they appear to be connected. We are actually extending the terrain surface with our additional features. The difference would be most obvious if we imagine the simulation of water flow over the terrain surface in each case. In the first case the buildings or dams could not interact with the flow; in our case these features, as part of the landscape, would divert the water. In a cadastral system based on our approach, the adjacency relationships between different portions could readily be determined merely by examining adjacent triangular elements. First, however, we should ask a little about what we might mean by a ‘3D Cadastre”.

Usually a cadastre is stored in two dimensions, defined as a parcel-based and up-to-date land information system containing a record of different kinds of claims on the land. However, today one land parcel may have one or more different owners-in multi-storey buildings, for example, different people can own parts of the same building and need to have access to some part of its observed exterior “surface”. This raises some difficult questions concerning what a cadastre “is” when extended beyond the two dimensional case. Various models are possible, starting from the simplest:

1 The cadastre is a set of unconnected polygons in two dimensions, with associated attributes. No attempt is made to specify adjacency.
2 Some form of “topology” is added to Model 1, guaranteeing connectivity and common boundaries and corners.
3 The properties of Model 2 are augmented, by specifying elevation information along the boundaries.
4 The properties of Model 3 are augmented by specifying rights “above” the property (in the air) or “below” (underground).
5 Model 4 is extended by providing a description of the (exterior) surface of the buildings, and assigning rights to portions of that surface in the same way as the previous models.
6 A complete 3D partitioning of the space occupied by the building is given (ignoring the air and ground rights mentioned in Model 4).

Most of these models present some problems. Models 1 to 5 are surface based, and hence the boundaries are observable, but may not catch all ownership situations. Model 6 may not be observable or measurable. Of course, underlying all this is the question whether the cadastre is monument-based or coordinate-based, as this will affect the viability of several of the models. Model 5 will be focused on here, as it follows the theme of our basic research. We will start with the base layer, and quickly summarize its properties.

Base Layer: The Quad-Edge Data Structure

“Make-Edge” and “Splice” (Figs. 1 & 2) are the two simple operations on the Quad-Edge structure, which is formed from four connected “Quad” objects, using the simple implementation of (Gold, 1998). Every Quad has three pointers. The Make Edge operator creates a new independent edge. The Splice operation, which is its own inverse, either splits a face loop and merges two vertex loops (upper to lower parts of Fig. 2) or else splits a vertex loop and merges two face loops (lower to upper). This operation suffices to maintain any connected graph on an orientable manifold, as is the case for CAD b-rep models, and TINs. Full details are given in Guibas and Stolfi (1985).

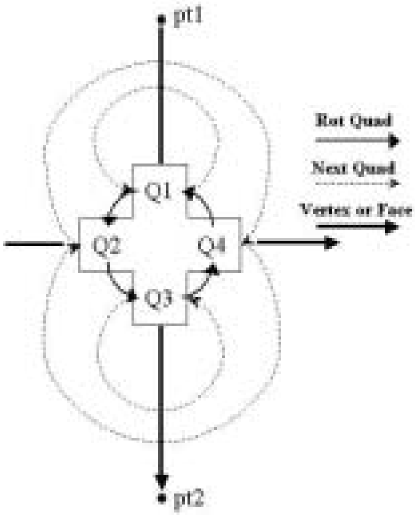

Figure 1. Make an individual edge

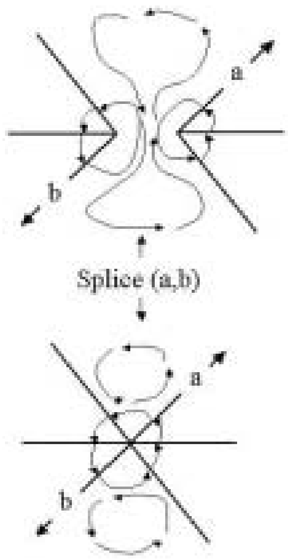

Figure 2. Splice

Middle Layer: Implementation of Euler Operators

The three basic Euler Operators are easy to implement using the Quad-Edge structure. The first, “Make Edge Vertex Vertex Face Shell” (MEVVFS) adds an edge, two vertices, one face and one shell (or body) to an empty model. Its inverse “Kill Edge Vertex Vertex Face Shell” (KEVVFS) removes them. (“MEVVFS” simply calls “Make-Edge.”) The second pair of Operators, “Make Edge Face” (MEF) and “Kill Edge Face” (KEF) are used for creating one new edge and face, and vice versa. In “MEF” two quads are used as inputs and an edge is used as input in “KEF”. Fig. 3 shows “MEF” and “KEF”. The third Operator pair, “Split Edge Make Vertex” (SEMV) and “Join Edge Kill Vertex” (JEKV), splits one edge into two pieces or merges two edges into one. It helps to reshape a face into triangle, as in Fig. 4.

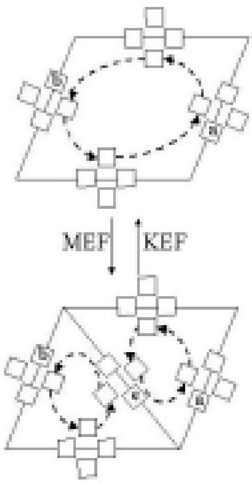

Figure 3 MEF ⇔ KEF

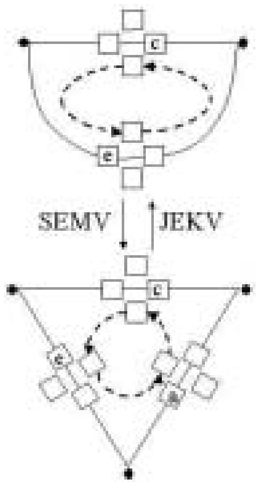

Figure 4. SEMV ⇔ JEKV

Upper layer: Creating a TIN model

In the TIN model, three operators are used: create the first triangle; insert a point; and swap an edge. Euler Operators are used to implement the triangulation functions, which are “Big Triangle”, “Insert Point” and “Swap”. The TIN model is started from a large triangle with three points enclosing the whole map area. Three different Euler Operators are used: “MEVVFS”, “MEF” and “SEMV” in Fig. 5. “MEVVFS” creates the first edge “e1”. “MEF” creates a new edge “e3”. “SEMV” splits edge “e3”.

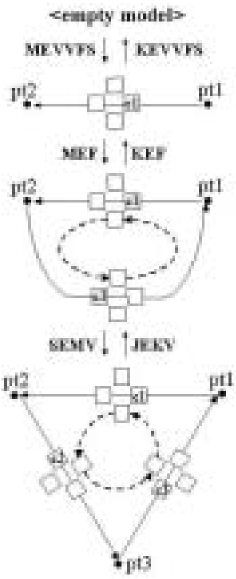

Figure 5 Big Triangle

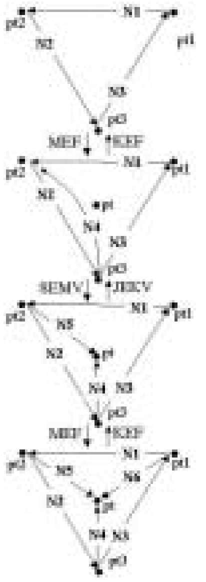

Figure 6 Insert Point

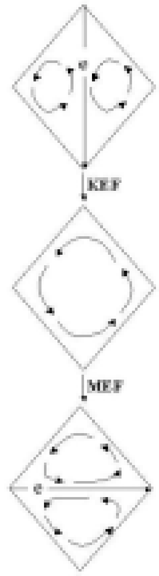

Figure 7 Swap

“Insert Point” is to create a new point in an existing TIN model. “MEF”, “SEMV” and “MEF” are used to insert the new point in the triangle as in Fig. 6. “Swap” exchanges an edge inside a quadrilateral, to generate well-shaped triangles inside the TIN model. Fig. 7 shows the swap operation using “KEF” and “MEF”.

Extended TINs

To make a hole in the TIN model we use another Euler Operator, as this validates the correctness of the topological structure, and the connectivity from outside to inside. “Make Edge Hole Kill Face” (MEHKF) has the same code as “MEF”, but it operates on two quads that are not in the same face loop, whereas “MEF” works on edges around the same face. “MEHKF” creates an edge between the two selected faces, as well as a curled face attached to this edge. Because the two selected faces are now removed, the total is one face less.

Fig. 8 shows the result after “MEHKF” (we look at the selected faces from “inside the model”, so they are in clockwise order). The connection of the edges will be:

- 1=>P =>5 =>6 =>4=> Q=> 2=> 3=> 1.

One face is killed and one hole is created, and the edge connectivity is preserved.
Fig. 9 shows the result after one more “MEF”. One new face is made and the connectivity of the edges will be:

- 1 => P =>5 =>R =>1 (A new face, the edges are in anti-clockwise order)
- and 3 => S => 6 => 4 => Q=> 2=> 3.

Fig. 10 shows the result of the final “MEF”. There are three faces inside the hole, and the connectivity is preserved and you can walk through the hole. Three more “MEF”s are used to reshape the inside faces into triangles.

We have shown that elementary Quad-Edge based Euler Operators are able to generate and modify the traditional TIN structure, extended as a general b-rep manifold (exterior surface). The assignment of attributes and rights to various portions of this surface potentially provides a visible and easily modifiable 3D cadastral system. Figs 11 to 14 show some pictures of constructing holes and bridges on the TIN model.

Beside bridges and holes, a building can be extruded from a TIN model. Fig. 15 and Fig. 16 show some buildings, and Figs. 17 illustrates a bridge connecting two buildings. (Of course, real buildings are not triangular!)

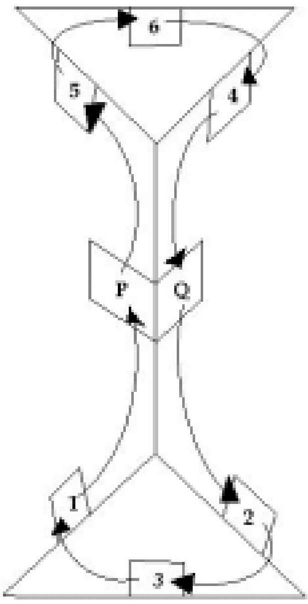

Figure 8 MEHKF

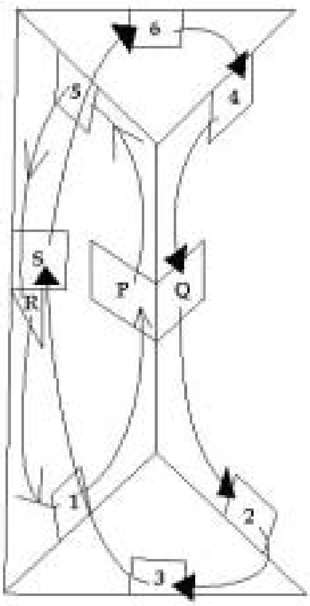

Figure 9 Second step MEF

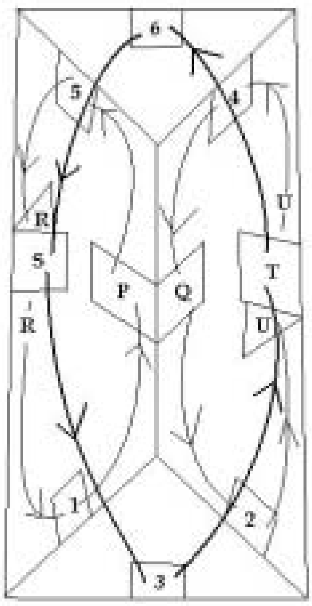

Figure. 10 Third step MEF

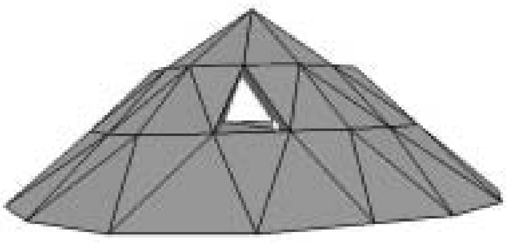

Figure 11 A hole on the TIN model

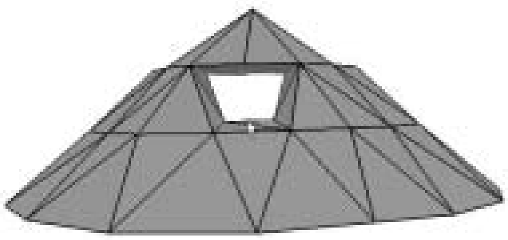

Figure 12 An enlarged hole by using Swap

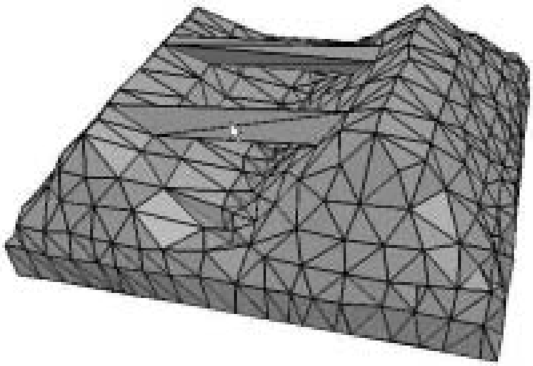

Figure 13 Two bridges on a TIN model

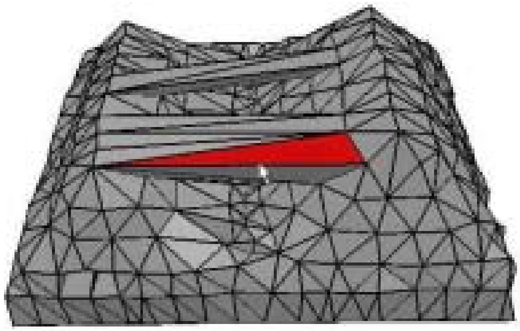

Figure 14 An enlarged bridge

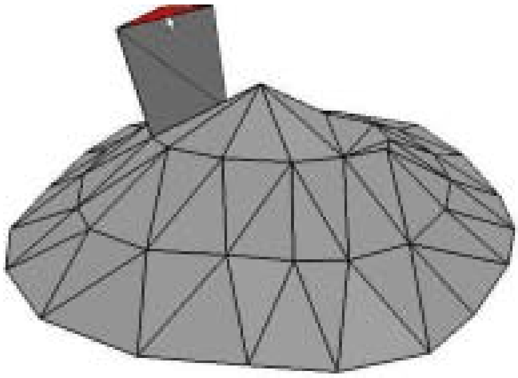

Figure 15 A building on TIN model

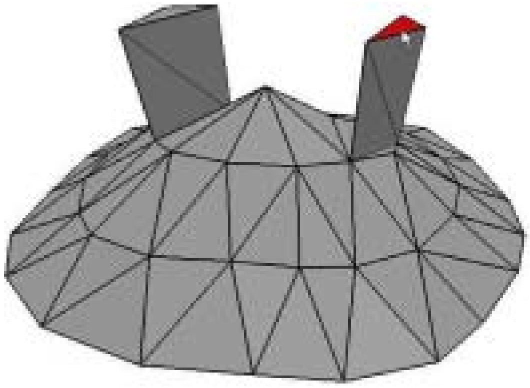

Figure 16 Two buildings

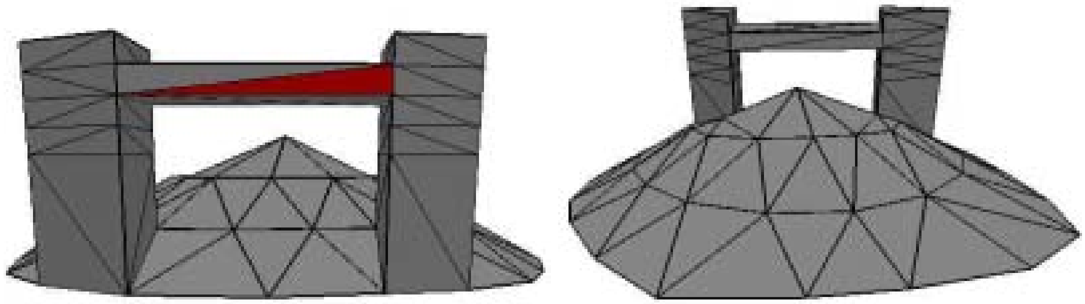

Figure 17 A bridge connects two extruded buildings on a TIN model

Conclusions

A three-level terrain modelling system has been described, based on Quad-Edges, Euler Operators and Triangulation commands. The benefits are: a) it is developed from the well-known TIN model; b) the addition of the CAD-type properties of Euler operators, including the guarantee of maintaining manifold connectedness, and the addition of features such as holes and bridges; and c) a greatly simplified implementation using Quad-Edges rather than the traditional winged-edge structure. The most difficult aspect of maintaining a cadastre is probably maintaining connectedness between spatial elements. This will be much worse in 3D. It seems that a validated b-rep model is a viable extension of the current 2D cadastral systems, which can be implemented by assigning rights or attributes to individual exterior faces.

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Further Readings:
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Rebeeca Tse is an MPhil student at the Department of Land Surveying and Geo-Informatics, The Hong Kong Polytechnic University, Hong Kong, China. Her Supervisor is Prof. Christopher Gold. After her bachelor's degree, she worked for one year as a GIS database analyst in a private firm. Her main research is interested in developing a topological structure for non-planar networks. It is related to the 3D representation of DTMs with underpasses or bridges for real networks. The Quad-Edge structure will be used to manage the 3D representation.

Christopher Gold currently holds the position of Professor in the same department, on leave from Laval University, Quebec. He first worked with TIN models in the 1970s, and has always been interested in spatial data structures. His PhD concerned spatial modelling in Geology, and more recently he has been very interested in Voronoi diagrams as explicit representations of spatial relationships, and has developed a variety of applications.

Figure Captions:

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