A REVIEW OF THE POTENTIAL APPLICATIONS OF VORONOI METHODS IN GEOMATICS.

Christopher M. Gold,

Chaire lndustrielle en Géomatique,
Centre de Recherche en Géomatique,
Pavillon Casault, Université Laval,
Québec, Qc Canada G 1K 7P4.
Telephone: (4 18) 656-3308,
Internet: CGOLD@VM 1 .ULAVAL.CA

Abstract

Occasionally, a new technique appears that has the potential to modify a discipline significantly. At the moment it appears that recent developments in Voronoi diagram methods in the field of Computational Geometry has the potential to change many of the ways we look at spatial problems in the discipline of geomatics or GIS.

This paper will start with an overview of some applications of Voronoi methods in spatial analysis. The simplest use the static point Voronoi diagram in the plane, the best known form. This may be extended in various ways, each with its own potential uses. For example, the diagram may be extended to three (or more) dimensions, or, still in two dimensions, the point generators may be replaced by man complex objects, e.g. line segments, with direct application to GIS type problems. The diagram may be constructed in other metrics, or using weighted generators. While the simple diagram represents the map zones associated with the nearest data point, equivalent diagrams may be constructed for the two nearest (or k nearest) data points - useful in various types of spatial queries. Finally, and perhaps the most interesting, these structures may be made dynamic - both in that they may be maintained through any set of insertions and deletions, and also because they may be maintained through point movement. This provides the basic tools for a truly interactive system, where the user and the map are involved in a "conversation". The implications of this are illustrated by describing interactive spatial decision support system requirements.

Résumé

Parfois, une technique nouvelle parait être en mesure de changer une discipline en profondeur. Présentement, il apparait que les derniers développements dans les méthodes de diagrammes Voronoi dans le domaine de la Géométrie Algorithmique ont le potentiel de changer de nombreuses façons de voir les problèmes spatiaux au sein de la discipline géomatique ou des SIG.

Cet article débutera par un survol de quelques applications des méthodes Voronoi en analyse spatiale. La plus simple utilise le diagramme Voronoi statique de points dans le plan, la forme la plus connue. Ceci peut être étendu de diverses façons, chacune d'elles ayant ses propres potentiels. Par exemple, le diagramme peut être étendu à la troisième dimension (ou plus) ou bien, en restant dans la deuxième dimension, aux générateurs ponctuels peuvent s'ajouter des objets complexes comme les segments de ligne, ayant une application directe aux problèmes typiques des SIG. Le diagramme peut être construit avec d'autres espaces métriques ou, en utilisant des générateurs pondérés. Tandis que le diagramme simple représente les zones de la carte associées au point de données le plus proche, des diagrammes équivalents peuvent être construits pour les deux (ou k) points de données les plus proches, ce qui est très utile dans divers types de requêtes spatiales. Enfin, ce qui est peut être le plus intéressant est le fait que ces structures peuvent être rendues dynamiques, parce que chacune d'entre elles peut être maintenue et pendant n'importe quelle combinaison d'insertions et de suppressions, et pendant le mouvement du point mobile. Ceci foumit les outils élémentaires pour un système réellement interactif, où l'utilisateur et la carte sont impliqués dans une "conversation". Les implications de tout ceci sont illustrés par la description des spécifications d'un système d'aide à la decision spatial et interactif.

1.INTRODUCTION.

This paper is about the rather abstract subject of how to model or represent space within a computer, although the main interest is in the new applications and flexibility provided by a new way of thinking about the problem. Nevertheless, many apparently unrelated problems may easily be handled with the Voronoi spatial model, whereas previously a variety of unrelated algorithms were developed. Some of these applications will be described, collected by diagram type.

2. STATIC POINT VORONOI DIAGRAMS.

These are well known in two dimensions, and a variety of applications have been developed. One very interesting application concerns pattern recognition of clusters of dots. Fig. 1 illustrates this: relatively circular Voronoi cells are in the interior of a cluster, whereas needle-like cells indicate a cluster boundary (see Ahuja and Tucetyan, 1989). Ogniewict and Ilg (1990) used the points forming the boundaries of images of objects on an assembly line to construct the medial axis transform or "skeleton" as a means of recognizing the object being viewed. Fig. 2 shows the equivalent (hand drawn) generation of the skeleton of human-like forms (taken from Blum, 1973). suggesting that this approach may be used for object recognition. For further discussion of these applications see Gold (1992b).

Another operation that may be usefully be based on the simple Voronoi diagram is surface interpolation from scattered data points. This has been discussed by Sibson (1982), Watson and Philip(1987) and others for the "natural neighbour" or "area-stealing" approach, where the query location is inserted as if it was a data point and the Voronoi area stolen from each of its neighbours used as a weghting in a weighted average technique. Fig. 3 shows a map generated from such data, along with the underlying Delaunay triangulation. Fig. 4 shows the influence function of one central data point surrounded by others. What is of particular interest is that, unlike other methods, a point is guaranteed to have no influence beyond the surrounding points. A similar approach may be used for line segments with known elevation attributes, and the surface may be constructed to be smooth, or just continuous (as in Fig. 5), or even completely diintinuous at these line objects. See Gold (1989) and Gold and Roos (1994) for details.

A further application was recently found as a rapid digitizing technique for forest mapping, where many points were rapidly digitized close to the inside of each polygon, and given its label (Fig. 6). The Voronoi diagram was then built (Fig. 7). and all edges between cells with different labels were then extracted to form the topologically complete forest map (Fig. 8). This has proved sufficiently precise and rapid for operational use (see Gold et al., 1994)

In three dimensions the point Voronoi diagram has been used to model crystal growth and their spatial relationships. In chemistry and astronomy they are used to model relationships among molecules and galaxies respectively - a dramatic change in scale!

4. OTHER STATIC VORONOI DIAGRAMS

Okabe et al. (1992) have given an outstanding survey of the various types of Voronoi diagrams and their applications, especially in the field of geography. The k-th order Voronoi diagram, which may be used in spatial queries of various types, consists of cells that are closest to any particular set of k points. For example, the 2nd order diagram may be used to ask which hospital patients should be sent to if any particular hospital is closed. (This assumes that patients will go to the nearest hospital - that is, all patients within the simple Voronoi cell for that hospital would normally go to the hospital at the centre of that cell.) Weighted Voronoi diagrams of various types (additive, multiplicative, or both) may be generated to represent territories of influence - whether for shopping centres or lions - when some individuals are more powerful than others.

The simple Voronoi diagram for points plus line segments is an interesting form of topology applied to the typical GIS polygon map. Fig. 9 shows that a complete set of spatial adjacency information is available for any combination of points, line segments, polygons or islands. This may be generated by static divide-and-conquer methods, or else by moving one point at a time within the previously-constructed part of the map, drawing a line segment behind (Gold, 1992a). This requires that the topological structure be dynamic, which opens up a great many applications possibilities, including the development of fully interactive systems that will be particularly valuable for the “what-if” type of queries of decision support systems.

4. SPATIAL DECISION SUPPORT SYSTEMS

There are various ways of classifying spatial decision support systems (Gold, 1993a), but we wish to distinguish primarily between global optimization methods, which are usually slow and difficult to interact with, and the "flight simulator" approach, which largely depends on the operator or decision maker to propose tentative actions, which are then evaluated by the machine. (An example is evaluating the consequences of sketching a new forestry road, and cutting all harvestable trees within some fixed distance: which road gives the best financial yield?) This assumes that the primary need is the ability to make small-scale decisions rapidly, and thus to have a spatial decision support system that really is interactive. This means that "what-if" modelling is possible, as with a spreadsheet: the user suggesting a scenario and the machine responding sufficiently rapidly that the decision-maker has not forgotten the context of his query. The objective is not to provide a globally optimum solution, but to allow the operator to make reasonable (and non-catastrophic!) decisions.

5. PLANE VERSUS BOAT: TWO APPROACHES TO INTERACTION WITH THE MAP

There are two main types of interaction with the supplied terrain information, that can most easily be visualized as the "plane" and "boat" approaches, and procedures involving combinations of these will be used in many GIS-type queries. For example, "plane" type digitizing has lines being drawn with fine disregard for the existing map "underneath", and only afterwards, with some relatively slow batch operation, are any spatial relationships established between the lines being drawn and the pre-existing map. (Many simple spatial queries are, by their nature, entirely of the "plane" type as they do not modify the spatial structure underneath.) By contrast, "boat" type digitizing has the cursor interact with the existing portions of the map, as it, being on the same level, would be in danger of collision with the land if real-time feedback were not available.

6. TRADITIONAL GIS SPATIAL OPERATIONS

This distinction is of importance for the development of interactive “flight-simulator” decision support systems. Traditional GIS queries may be of two types: those that do not, and those that do, modify and rebuild any topology or spatial relationships. Examples of the first include totals of polygon areas, or all polygons adjacent to some specified polygon type. Whenever the query involves the three classic spatial GIS operations, however, then topology modification or construction is needed. These operations are usually quoted as: polygon reclassify and merge; buffer zone or corridor construction; and polygon overlay. These have been considered the “difficult” GIS operations because, as traditionally performed, they all involve rebuilding the polygon structure - and this is a global operation, because it depends on detecting all the intersections of potential boundary segments. For many years this was the central operation, with lots of special cases, and very much a trade secret.

This operation was an expensive global process, which became the bottle-neck and focus point of GIS processing, because of the absence of a meaningful operational definition of the term “local” - thus enforcing a complete rebuild after every topological change. Thus the “plane” mode1 had no interaction with the pre-existing map structure, and after all the lines were added the global polygon rebuild was required. In addition, note the emphasis on “polygon” - most activities were for regional analysis. For data in the form of stream networks, or independent data points, there was less, or no, topological validation. Islands were not readily associated with their containing polygons, as no line intersections were observed. The model of space was inadequate for flexible spatial operations.

7. A SPACE-FILLING, DYNAMIC SPATIAL MODEL

So much for the "plane" approach. Enter the "boat" model of space. Here the operator, pen or cursor is part of the map world, and interacts directly with it. This allows local modifications of the map without global reconstruction. This is achieved by rejecting the line-intersection model of space and replacing it with the proximal model: all space is always fully occupied, and consists of tiles or bubbles around each map object (point or line segment). This is also called the Voronoi model. Objects that were disconnected in the line-segment model may well be neighbours ii. the Voronoi model, if their bubbles touch, A bubble is defined as enclosing all locations closer to that map object than to any other. The "boat" representing the operator or cursor is also a map object with its own bubble, and thus has a set of neighbours with which it interacts. This permits many operations to be performed in a local fashion, rather than globally, as well as new operations not even feasible with global procedures.

Using the Voronoi or bubble dynamic spatial model lines may be drawn, deleted or intersected, and points may be created, deleted or moved directly by the user - as in a fully interactive digitizing system. From the proximal model of space that is used, it is not necessary to draw on top of, or point precisely at, an existing object in order to select it for use (e.g. to snap a line to it). Merely pointing nearer to that object than to any other will suffice. (This is the same as pointing anywhere within an object's bubble.) This follows pretty closely the human gesture when selecting something on a map, and some work has been done on relating the Voronoi spatial model to human perception of spatial relationships (Gold, 1992b). Polygon building in an incremental manner is thus greatly simplified. The usual difficulties with the 'polygon merge" operation are handled in a well-behaved local fashion, in real-time if necessary. The other classic single-coverage operation, the construction of corridors or buffer zones around selected features such as roads, also becomes a trivial operation, since all corridor boundaries must fall within the proximal zone (Voronoi region) of the point or line segment component of the feature. No intersection tests or other global operations are required. The last standard GIS operation, "polygon overlay", may be performed by tracing each segment of coverage A through coverage B, producing the required colours and intersections of each line segment to form the final combined coverage C, which is itself built as a Voronoi diagram. This step is repeated tracing B through A to form a complete polygon overlay, but for many applications this second step is not really required.

8. OPENING UP THE FUTURE

So much for standard polygon GIS operations. Operations on incomplete polygon structures, and those having apparent dangling segments, are readily handled - either by flagging them as errors, or else permitting them, as required. Network structures are managed with exactly the same tools. Interpolation problems, both with point data and with linear data elements or faults, follow the same path also - indeed most of these techniques were originally developed for terrain modelling purposes. Many restrictions due to the old topological structures are alleviated, new combinations of queries become feasible, and dynamic interactive operations may be developed.

Another class of operations is not normally considered feasible within a GIS: the real-time interaction of the user with the digital map - the "virtual reality". As we have seen with the interactive digitizing example, the "boat" approach allows the user to query or respond to the existing map, as well as to modify it. Two immediate applications come to mind - but there will be many more. The first suggests a robotics application, where the "boat" or cursor may be given various rules about topography, slope, obstacles or roads and will attempt to determine a feasible path, perhaps with human assistance from time to time. (Static Voronoi diagram "skeletons" are already used for this, but merely to provide a feasible graph through a maze.) An example is given in Fig. 10, where boat "A" may be guided automatically because of the well defined set of neighbours to its Voronoi "bubble". Its optimal channel is probably along the Voronoi boundaries, or skeleton, e.g. within the harbour entrance. Boat "B" is shown navigating through a set of depth soundings, and the introduction of its Voronoi bubble permits the direct local interpolation of the channel depth. The integration of polygons, networks and interpolation within the same structure permits many similar dynamic applications.

The second use of the interactive "boat" approach concerns our original "flight-simulator" view of spatial decision support systems. Here the operator is making local tactical suggestions (in the forestry case perhaps a new branch of a forestry road), and the machine is expected to evaluate the proposed action. In the forestry case, perhaps a buffer zone is to be constructed about the branch road, overlaid with the forest stand map, and a harvest summary given. Given the local nature of the Voronoi method, this is a rapid operation, and does not require the rebuilding of large portions of the map. The response to the user must be fast - a few seconds - because the nature of the "flight simulator" is to "fly" the problem, adjusting things and trying again, until two things happen: the operator gets a "feel" of the way the system is responding, and a reasonably satisfactory action (e.g. the forest road selection) is accepted. In many operational settings this is preferable to a somewhat artificial "optimal" solution, that nevertheless appears unreasonable in the field.

9. CONCLUSION: THE REST IS HISTORY

The final aspect of our research follows directly from the incremental nature of the method. This means that actions were performed in a particular sequence, and these may be recorded, played back, and modified. This introduces the idea of "the map as a movie". A good example again is forest mapping. Given the current forest inventory map, it is necessary to update it with the latest information about forest roads, cut areas, etc., on a regular basis. Thus the original map is modified. What happens, however, when it is required to evaluate the history of a particular site? The whole original map has to be compared with the updated one, creating problems, and this is compounded if a long-term history of various uses is required, e.g. IO evaluate reforestation strategies.

In the incremental approach, any changes to the map are merely appended to the previous information, together with relevant date information. The "tape recording" of the map is then played back to any desired date. for evaluation. Comparisons may then be made at any location. as to what changes occurred, and when. As with the whole Voronoi approach, this is event-driven over time, not a series of snapshots, removing much redundancy. For further details see Gold (1993b).

This brief review has been intended to sketch out the general implications of a new, more flexible approach to modelling space and time. It is object-oriented, rather than coordinate-oriented, and we are only just beginning to enumerate its applications.

ACKNOWLEDGEMENTS

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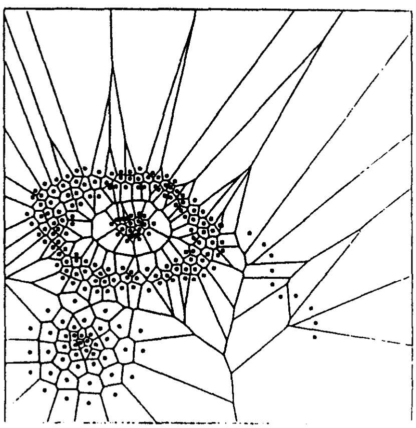

Figure 1. The Voronoi diagram as art aid in poinr cluster analysis.

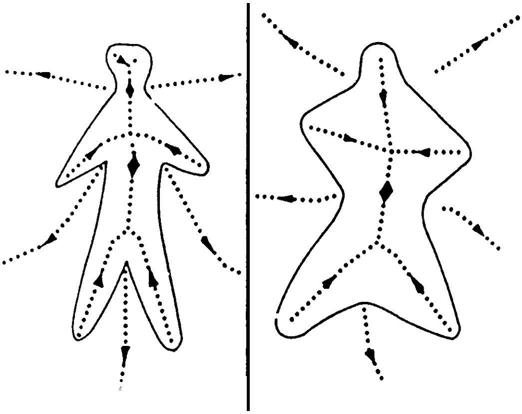

Figure 2. Two anthropomorphs and their skeletons. From Btum (1973).

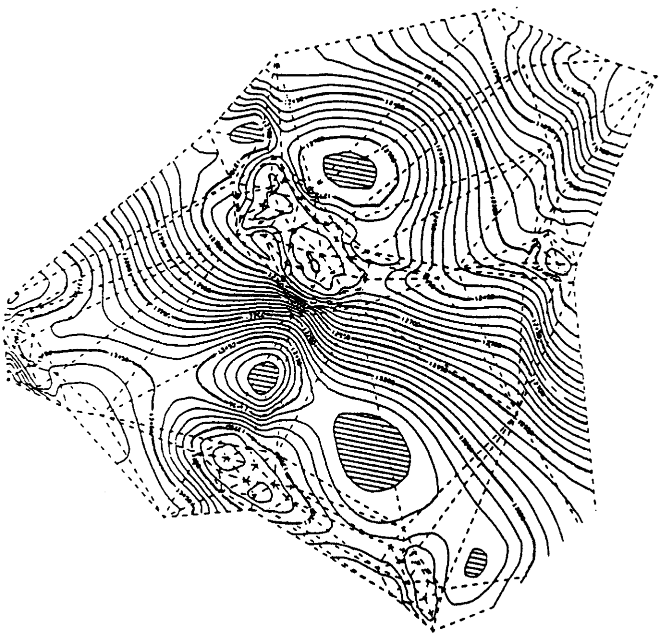

Figure 3. A geological contour map and the Delaunay triangulation.

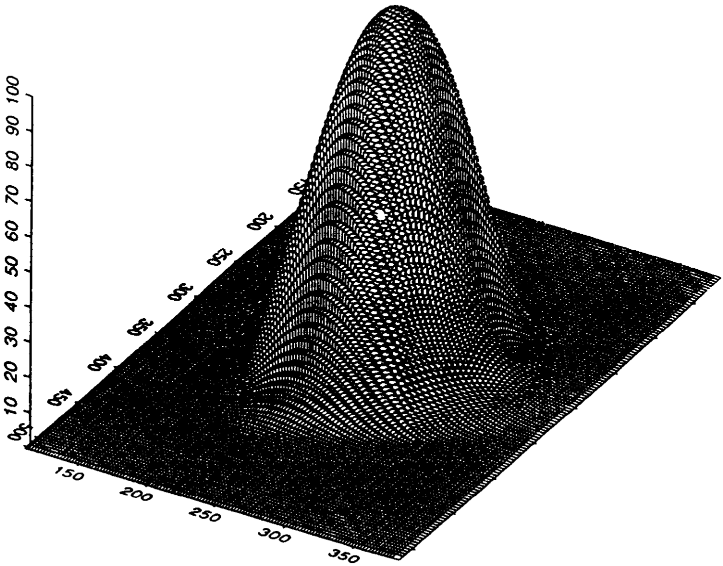

Figure 4. The smoothed area-stealing influence function.

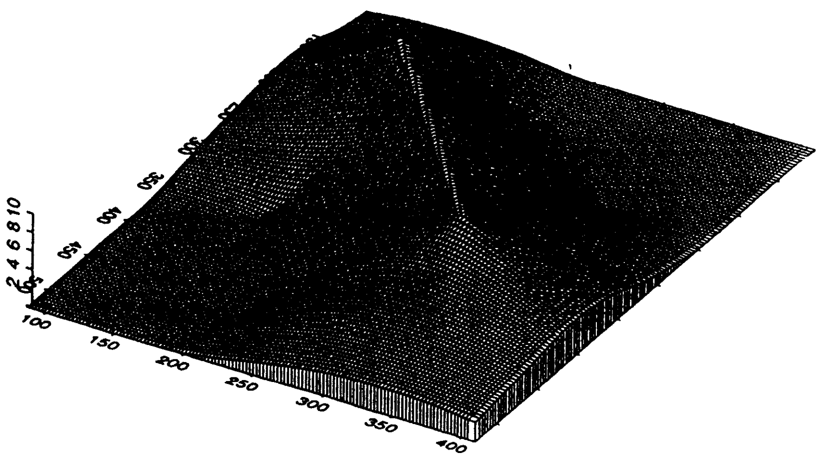

Figure 5. The area-stealing influence function for a line segment.

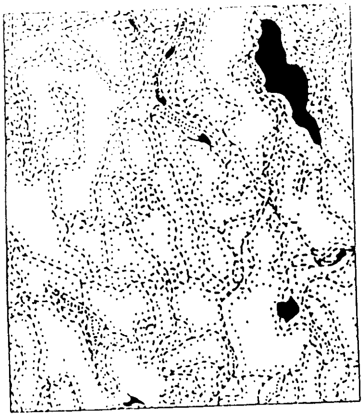

Figure 6 Digitized points inside forest stands.

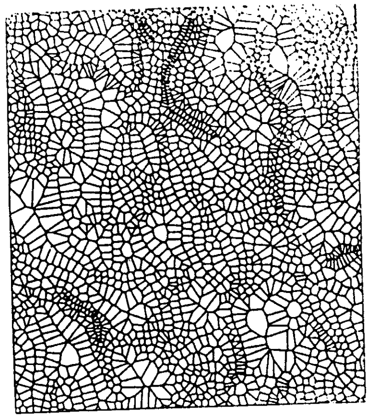

Figure 7. The Voronoi cells of Fig. 6.

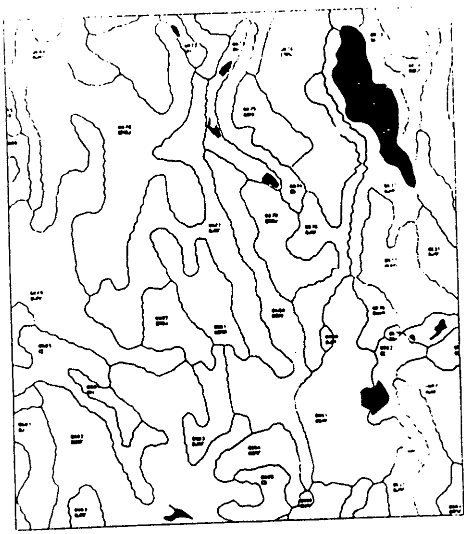

Figure 8. Boundaries extracted from Fig. 7.

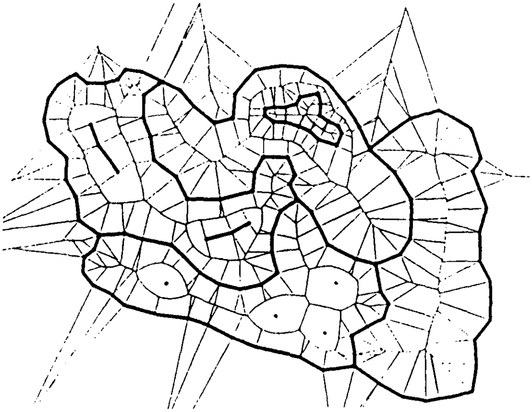

Figure 9. The Voronoi diagram for points, lines and polygons

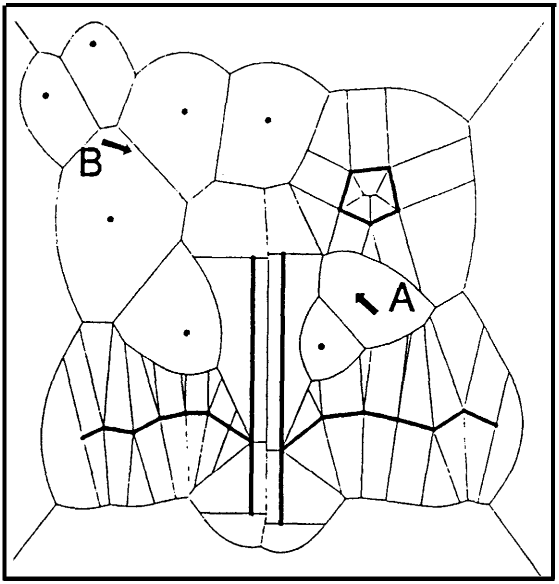

Figure IO. Voronoi mcrhods for navigation.