A proposed connectivity-based model for a three-dimensional cadastre

Rebecca, O.C. Tse and Christopher Gold

Department of Land Surveying and Geo-Informatics
Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
Tel: (852) 2766-5955; Fax: (852) 2330-2994
Rebecca.Tse@polyu.edu.hk, christophergold@voronoi.com

Abstract

This paper proposes a new approach to create a three-dimensional (3D) cadastre using a surface-representation model. It starts by describing a variety of possible approaches, in order of increasing complexity, and then selects one that gives significantly increased flexibility with only slightly increased difficulty in implementation.

Techniques from Computer Science (Computational Geometry) and Engineering (Computer Aided Design, or CAD) as well as from Geographical Information System or GIS (TIN -Triangulated Irregular Network modelling) are used. Starting with the Quad-Edge data structure as the basis, a simple implementation is described: this approach is often used for TIN models. We then show a simple subset of CAD-type Euler Operators and describe how they may be implemented with Quad-Edges. A simple TIN model may be built using these Operators. However, the addition of one more Euler Operator permits the creation of bridges and holes in the triangulated surface. This gives sufficient tools for representing complex man-made objects.

A three-dimensional cadastral system based on this surface model is proposed. As in a two-dimensional, parcel-based system, property rights are associated with the visible surface, together with “interior” and “exterior” rights corresponding to “above ground” and “below ground” in current systems. This is simple to implement, and flexible enough to be useful.

1. Introduction

In this paper a variety of possible models for a three-dimensional cadastre is discussed, in order of increasing complexity. Then one is selected that appears to have increased flexibility with minor difficulties in implementation. This is a boundary representation (B-Rep) model, where buildings are considered to be extensions of the terrain, and property rights are assigned to portions of the visible exterior surface. “Boundary” here refers to the two-dimensional boundary of a three-dimensional solid, rather than the one-dimensional boundary of a two-dimensional parcel.

In order to implement this model, research done in Computer Science and Computer-Aided Design (CAD) was explored, as well as the well-known Triangulated Irregular Network (TIN) model used in GIS. B-Rep based CAD systems permit the extrusion of regular shapes, and the creation of bridges and holes, by using Euler Operators, and a simplified set of these is readily applicable to TINs. Traditional Euler Operators use a complex "Winged-Edge" data structure. We can simplify this greatly by using the "Quad-Edge" data structure familiar to some branches of Computer Science, and already used in some TIN models. These are described in more detail later.

The paper therefore starts with a brief description of the Quad-Edge structure, and follows it with a description of some Euler Operators, and how they may be constructed using Quad-Edges. Then the operators are used to implement a simple triangulation structure, and the same operators can be used to construct simple buildings. The addition of one more Euler Operator permits the creation of bridges and tunnels, as well as more complex buildings, thus providing a simple extension to the TIN that provides a flexible surface model that can be used to assign property rights.

2. Definition of cadastre

Cadastral data are usually stored in two dimensions, for example following the definition: "A Cadastre is normally a parcel-based and up-to-date land information system containing a record of interests in land (i.e. rights, restrictions and responsibilities). It generally includes a geometric description of land parcels linked to other records describing the nature of the interests, ownership or control of those interests, and often the value of the parcel and its improvements." (Gérard, 1999). A new definition of a cadastre was described by Kaufmann (1999): "Cadastre 2014 is a methodically arranged public inventory of data concerning all legal land objects in a certain country or district, based on a survey of their boundaries." This introduced a more general definition of a land object. Land can be viewed as a discrete object with homogeneous conditions inside its boundaries. This definition better matches representation of the real world, where one land parcel may have one or more different owners. Especially for multi-storey buildings, many people can own parts of the same building and need to have access to some part of its visible exterior. Cadastral systems are described in more detail in Williamson (1995) and Henssen (1995).

These definitions raise some difficult questions concerning what a cadastre is when extended beyond two dimensions. Various models are possible, some are listed below:

1. The cadastre is a set of unconnected polygons in two dimensions, with associated attributes. No attempt is made to specify adjacency. This is the simplest case, parcel overlap is possible.
2. Some form of "topology" is added to Model 1, guaranteeing connectivity, common boundaries and corners. Most polygon-based GISs would support this model.
3. The properties of Model 2 are augmented, by specifying and including elevation along boundaries.
4. The properties of Model 3 are augmented, by specifying rights "above" the property (in the air) or "below" (underground). This usually becomes explicit when subsurface rights are involved.
5. Model 4 is extended, by providing a description of the exterior of buildings, and assigning rights to portions of the surfaces in the same way as the previous models. This is the model proposed in this paper.
6. A complete 3D partitioning of the space occupied by buildings is given, ignoring the air and ground rights mentioned in Model 4. Here it is necessary to implement some type of solid model, with interior boundaries, some of which would not be visible.

Most of these models present some problems. Models 1 to 5 are surface based, and hence the boundaries are visible, but they may not catch all ownership situations. Model 6 may not be observable or measurable. Of course, underlying this is the question of whether the cadastre is monument or coordinate-based, as this will affect the viability of several models. If monument-based we would need to be able to observe some corners. If coordinate-based this would not be necessary, but the definition may not correspond to reality. Our research is based on Model 5, as this seems to be a reasonable extension of previous practice.

3. Topology vs. database

Topological properties are invariant under continuous distortion. "Neighbourhood" is a topological property because two regions will always be next to each other no matter how a map is distorted. "Enclosure", which relates an interior region to an outer region enclosing it, as well as "Connectivity" which relates a line to a connecting line, are other topological properties. These three are the most important topological relationships. Therefore the concern is mostly with the connectivity of a 2D manifold (surface) embedded in 3D space, which separates the inside of the object (the polyhedral earth) from the outside (air or water). This manifold is formed from connected planar elements – usually rectangles or triangles. The focus is on triangular elements, as they are the most flexible and the most widely used.

The key technologies involved are the traditional TIN terrain model and some elements of CAD systems (e.g. Zeid, 1991), implemented using data structures developed in Computational Geometry. While individual triangles can be stored in a relational database, as with conventional shape files, they would still need modification tools. Basic traversal of triangulations would be extremely inefficient. Other operations, that require the "intersection" of different "topological spaces" (e.g. pipelines with land ownership parcels), will still need to be calculated as required. The choice of a network-based topological model, or a model based on individual polygons stored in the database and matched as required, is still unclear.

However, the complete TIN model of our land and buildings may be too large to hold in memory and it is undesirable to separate the cadastral database from the surface model. This problem is fundamental to all GIS methods and we do not claim to have eliminated it. However, our concerns are similar to those of database experts: how to guarantee the integrity of the database - particularly the topological structure. It is not enough to have a valid data model or structure, one needs to be sure that the "transactions" are complete. For this, the concept of "Euler Operators" as used in b-rep (surface, or manifold) models used in CAD systems have been tried. These operations on a completely connected b-rep are guaranteed to preserve the mesh connectivity because they preserve the Euler (or Euler-Poincaré) formula, (see Lee, 1999). Constructive Solid Geometry (CSG) models will not be used, where independent solids are combined to give the desired result. Nor will "non-manifold" CAD systems be discussed (e.g. Lee, 1999), although these could possibly provide the same level of "transaction validation" in the future. Instead, Euler Operators are implemented, using the Quad-Edge structure (Guibas and Stolfi 1985), which is both simple and elegant. Recent work (T. Merrett, personal communication) suggests that "long transactions" could be designed to handle a variety of queries on the Quad-Edge structure in a relational database and in a relatively efficient manner.

4. Boundary representations vs. a non-manifold model

Manifold models are similar to the 3D graphics modelling systems used to create dinosaurs, etc., for computer games. In two dimensions these would be equivalent to "shape" file models where each object is discrete and has a well defined inside and outside. Here the connected topology is simply the sequence of points forming the boundary. In 3D this would be equivalent to the surface mesh – probably a triangulation – that separates inside from outside. There is no explicit relationship between individual objects – in the dual sense there are only two nodes with two colours (solid/air). 2D connected topology, as in a choropleth map, makes no such assumption and in this case the boundaries are one-dimensional edges. 3D connected non-manifold topology is similar: the boundaries are formed from triangulated surfaces separating pairs of solids with different colours. Thus inside and outside are only relevant to a single volume element and a dual tetrahedral network defines the spatial adjacency relationships. We have restricted ourselves to a single volume element – the polyhedral earth, which can be modelled in the same way as imaginary dinosaurs - and therefore do not attempt to partition this solid further.

As stated by Mantyla (1981, 1988), "In a boundary representation model (b-rep) an object is represented indirectly by a description of its boundary. The boundary is divided into a finite set of faces, which in turn is represented by their bounding edges and vertices." According to his definition, b-reps are best suited for objects bounded by a compact (i.e. bounded and closed) manifold.

In 1988 Weiler proposed non-manifold operators to manipulate topological data in non-manifold models. They were not based on the basic Euler-Poincaré formula, although some of the topological relationships could still be generalized by a new formula (see Masuda et al. 1990). However this formula is complicated and more operators are needed. It may be valuable to extend the b-rep with Euler Operators into a non-manifold model in the future, but we have not attempted to do so here, as the concept of an "exterior surface" seems sufficient.

A further consideration with using non-manifold models for cadastral purposes is that there is no defined inside and outside. There is no guarantee that any desired feature (e.g. an interior boundary) is observable - which is required for a monument-based cadastre. The 2D equivalent is Model 4, where rights are assigned above and below the surface and interior properties may be defined as rights associated with a visible exterior. This may not handle all cases, but it validates the concept of accessibility of parcels, and may suffice for most applications.

5. The Quad-Edge data structure

In the current work the CAD-type b-rep structure and Euler Operators are used to create a connected TIN model with holes (bridges or tunnels). First, the basic properties of the Quad-Edge data structure of Guibas and Stolfi (1985) are described, followed by a subset of CAD-type Euler Operators that suffices for building TIN models. These are easily implemented using Quad-Edges. The Euler Operators will then be used in a simple TIN model. The addition of one more Euler Operator allows the construction of bridges and holes, and the surface definition of the 3D cadastral system is shown.

The Quad-Edge data structure is used to implement the set of Euler Operators because it allows navigation from edge to edge in any connected graph on a surface. “Make-Edge” and “Splice” are the two operations used to maintain the Quad-Edge structure of points and edges. Quad-Edges are formed from four connected “Quads”. Make-Edge (Fig. 1) creates a new edge connecting two vertices, while Splice (Fig. 2) is used to merge two vertices by splitting the previous common face, or else to merge two faces by splitting their common vertex. Our implementation of Quad-Edges has three fields:

- N – link to next Quad “Next” anticlockwise around a face or vertex
- R – link to next ¼ Edge “Rot” anticlockwise around the four Quads
- V – link to vertex (or face)
- Each Quad has an “origin” vertex (or face) identified by V, and a “destination” vertex identified by R. R. V.

Algorithm for Make edge (Fig. 1)

Make edge for points 1 and 2

- 4 Quads will be created *[Q1, Q2, Q3 & Q4]*
- 4 Quads will link four parts in anti-clockwise order *[using R]*
- Q1, Q2, Q3 and Q4 point to each other in anti-clockwise order
- Q1 and Q3 point to pt1 and pt2 *[V points to pt1 and pt2]*
- Q2 and Q4 point to the left or right faces

Algorithm for Splice (Fig. 2)

Splice (a, b: Quad) → a & b: input Quad-Edges

- get neighbour edges : Alpha & Beta (Guibas & Stolfi, 1985)
- reconnect the four pointers

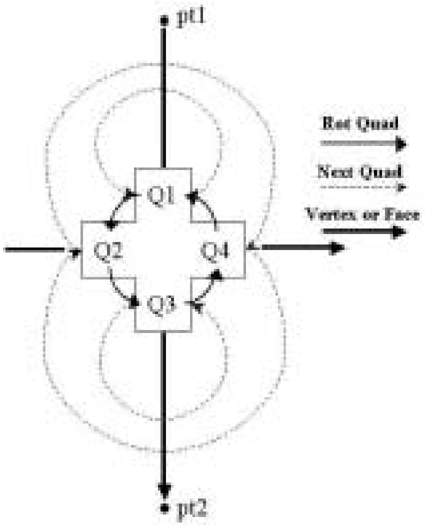


Figure 1 Make edge

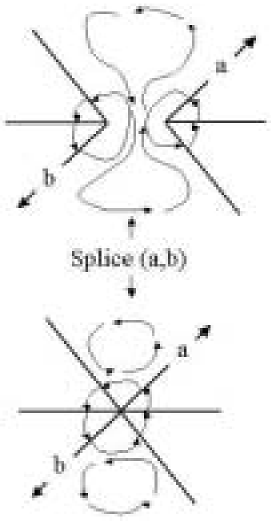


Figure 2 Splice

6. Implementation of Euler Operators using the Quad-Edge data structure

The Quad-Edge structure is used to implement the three Euler Operators. Other Euler Operators have been tested as well, but these three suffice to create TIN models. This set of Euler Operators is used to simplify the assumption that we are always working with a triangulation (or, more specifically, that no faces have holes). Table 1 gives a brief summary of the Euler Operators that we used in our research.

The Euler Operator	The entities are modified
MEVVFS (KEVVFS)	Make (kill) one edge, two vertices, one face and one shell (or body)
MEF (KEF)	Make (kill) one edge and one face.
SEMV (JEKV)	Split (joint) one edge, make (kill) one vertex.
MEHKF (KEHMF)	Make (kill) one edge and one hole, kill (make) one face.

Table 1 The Euler Operators

6.1. MEVVFS ⇔ KEVVFS

To start a model, a new object must be created. Make Edge Vertex Vertex Face Shell (MEVVFS) is used to create a body (shell), a face and an edge with two vertices. Kill Edge Vertex Vertex Face Shell is used to remove the object and return to the empty model.

Algorithm for MEVVFS and KEVVFS

MEVVFS with pt1 and pt2 as two input points

- e = MakeEdge *[e is the new edge between pts 1 and 2]*

KEVVFS with e as one input edge

- Remove the body, face and edge with two vertices *[return to an empty model]*

6.2. MEF ⇔ KEF

Make Edge Face (MEF) and Kill Edge Face (KEF) are used to split a face by adding an edge and a face (MEF), or to merge two faces by killing an edge and a face (KEF). In MEF we need to input two quads as parameters to make a new face. In KEF we need to give an edge as a parameter for removing the edge. The related face will be destroyed as this edge is removed. Fig. 3 shows the procedure for MEF and KEF.

Algorithm for MEF and KEF (Fig. 3)

MEF with a and b as the two input quads:

- e = MakeEdge (a's origin to b's origin) *[e is the new edge]*
- Splice a and e edges *[connects e with a]*
- Splice e's Opposite and b edges *[connects e with b]*

KEF with e to be removed

- Splice a and e *[disconnects a and e]*
- Splice e's Opposite edge and b *[disconnects e and b]*
- Remove edge e

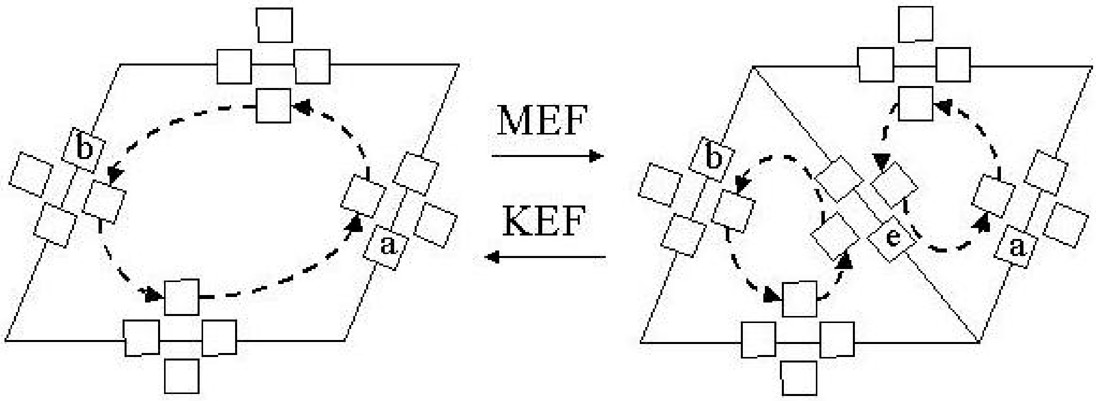


Figure 3 MEF ⇔ KEF

6.3. SEMV ⇔ JEKV

Split Edge Make Vertex (SEMV) and Join Edge Kill Vertex (JEKV) are used to split one edge into two pieces. This procedure adds or removes a point on a line. It is useful for creating triangles without creating a dangling arc; also it helps to reshape a face into a triangular shape. Fig. 4 shows the procedure of SEMV and JEKV.

Algorithm for SEMV and JEKV (Fig. 4)

SEMV inputs e (1 edge) and pt (1 point)

- Splice e and c *[disconnects e and c]*
- Change e's destination to pt
- a = MEF (e, c) *[a new edge a is created from e's destination to its origin]*

JEKV inputs e (1 edge)

- Splice e and a *[disconnects e and a]*
- Splice a and c *[disconnects a and c]*
- e's destination = c's origin *[lengthens e's edge to c's origin]*
- Splice e and c *[connects e and c]*

- remove vertex pt and edge a *[releases one point and one edge]*

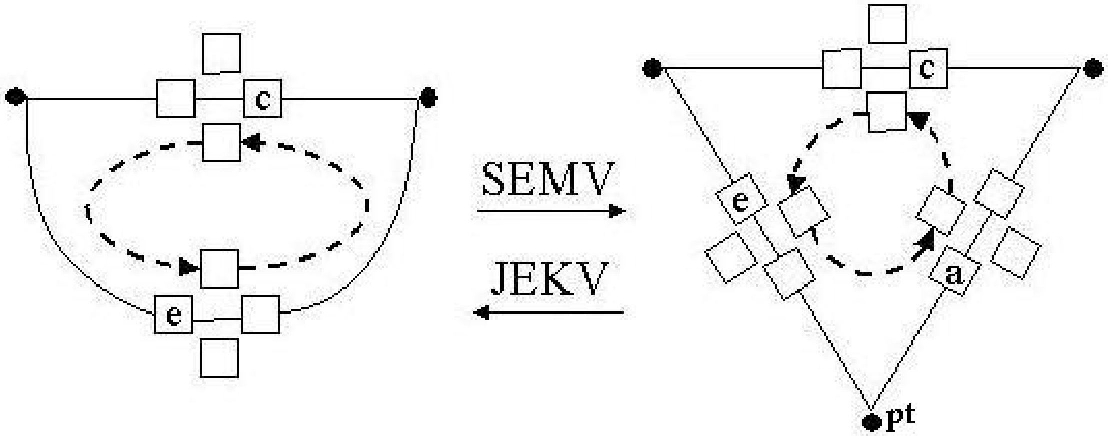


Figure 4 SEMV⇔JEKV

7. Implementation of the TIN model using Euler Operators

7.1. Big Triangle

The TIN model is started from a large triangle with three points. Fig. 5 shows the procedure for creating a big triangle.

7.1.1 Algorithm for Big Triangle (Fig. 5)

The big triangle inputs three vertices pts 1, 2 and 3

- e1 = MEVVFS(pt1, pt2) *[e1 is created from pt1 to pt2]*
- e3 = MEF (e1, e1's opposite Quad) *[input the same edge e1 with a different direction, e3 is created and a face is created from pt2 to pt1]*
- e2 = SEMV (e3, pt3) *[e2 is created and the face is reshaped into a triangle]*

7.2. Insert Point

Insert point is a procedure that will be used in the TIN model. First "Walk" through the TIN model to locate the triangle which contains the newly inserted point and return the nearest edge in this triangle. In the TIN model the whole surface is formed by triangles, therefore a new point will be inserted into an existing triangle. Fig. 6 shows the procedure for inserting a new point.

7.2.1. Algorithm for Insert Point (Fig. 6)

Insert Point input a new point (pt)

N2 is the nearest edge to the point *[return from Walk]*

- N4 = MEF (N3, N2) *[a new edge N4 is created between pts 3 and 2]*
- N5 = SEMV (N4, pt) *[split edge N4, an edge N5 and a point pt are created. This reshapes the face into triangle]*

- N6 = MEF (N1, N4) *[an edge N6 is created to split one face into two. The edge runs from pt1 to pt]*
  [After inserting a new point, two faces will be created, as one face already belongs to the existing triangle.]

7.3. Swap

Swap's function is to swap edges inside a quadrilateral, to generate well-shaped triangles inside the TIN model. The "in-circle" test is used to test the triangle if required, and is used the Swap operator to change edges. Fig. 7 shows the operation of Swap. Recursive testing and swapping is sufficient to generate a "good" triangulation (e.g. Delaunay).

7.3.1 Algorithm for Swap (Fig. 7)

Input to swap: (e) an edge needs to be changed

- Kill edge e using KEF
- Make a new edge e using MEF

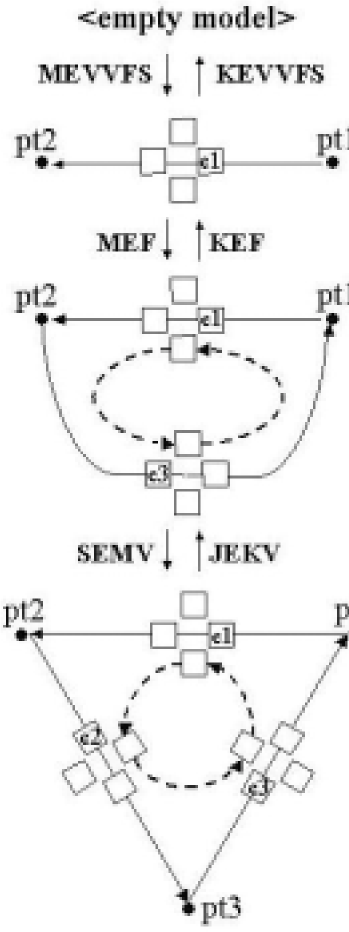


Figure 5 Big Triangle

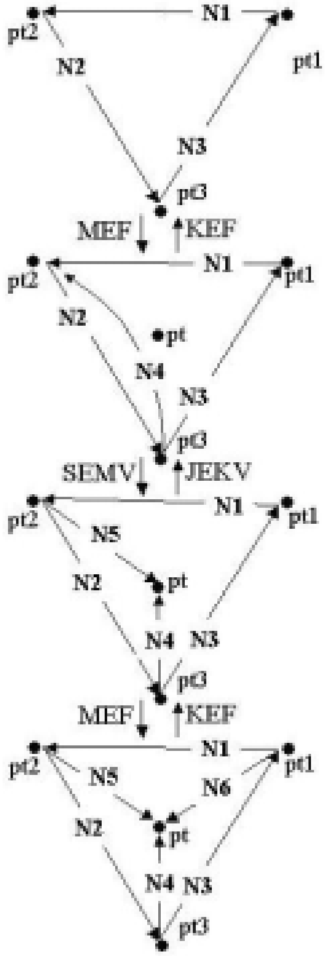


Figure 6 Inert Point

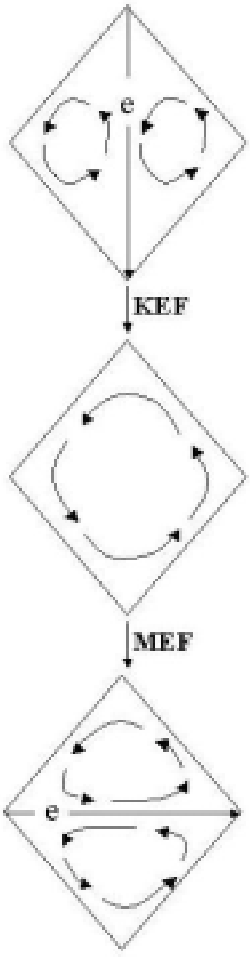


Figure 7 Swap

8. Additional Euler Operators for creating holes

The Euler Operators described above are used to modify surfaces, for example by adding simple buildings. However, for the insertion and deletion of holes in TIN models a new Euler Operator is needed. The new operator Make Edge Hole Kill Face (MEHKF) has the same code as MEF, but MEF operates on two edges of the same face while MEHKF operates on separate faces. MEHKF creates an edge between two selected faces, as well as a curled face attached to this edge. Because the original two selected faces are now removed, the number of faces is one less.

In Fig. 8, an existing TIN model, two triangles A and B are selected. Their edges are ordered anti-clockwise on the outside surface. (In Fig. 9 we are looking from the inside.) They are used to create a hole, and observed from anywhere inside the hole (but outside the solid interior of the object) the connectivity of edges is anti-clockwise. This preserves the connectivity of inside and outside edges. The orders of the three edges inside two triangles are:

- 1 => 2 => 3 => 1 of triangle “A”
- and 4 => 5 => 6=> 4 of triangle “B”

which are shown in Fig. 9 (seen from the inside). A new edge will be made between these two triangles and a hole will be created.

Fig. 10 shows the picture after the first step – MEHKF. The connection of the edges will be:

- 1=>P =>5 =>6 =>4=> Q=> 2=> 3=> 1.

One face is killed and one hole is created: the edge connectivity is preserved. Fig. 11 shows the result after performing one more edge and face MEF. One new face is made and the connectivity of the edges will be:

- 1 => P =>5 =>R =>1 (New face)
- and 3 => S => 6 => 4 => Q=> 2=> 3.

Fig. 12 and 13 show the result of the final MEF and the outside view of the hole: that is from the inside of the solid object! One new edge and face are made. There are three faces inside the hole, but the connectivity is preserved and you can walk through the hole while staying on the exterior of the manifold. Three more MEFs are used to split those faces inside the hole into triangles.

We have therefore shown that the elementary Quad-Edge based Euler Operators are able to generate and modify the traditional TIN structure, extended as a general b-rep manifold. The assignment of attributes and rights to various portions of this surface provides a visible and easily modifiable 3D cadastral system.

The same procedure can be used for creating a bridge between two triangles. Figs. 14 and 16 show a hole in a TIN, and bridges on a TIN model. “Swap” is used to enlarge the hole and the bridge in Figs. 15 and 17.

The same idea can be used to create a building and other complex structures. A triangle on a TIN model is selected and a specified height is given to extrude a building. Figs. 18 and 19 show a building on a TIN model, and a bridge connecting two buildings.

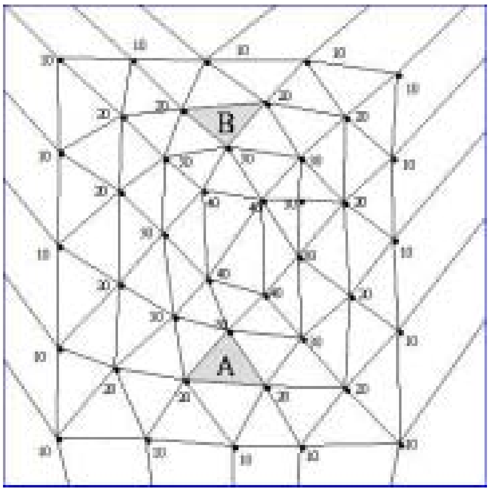
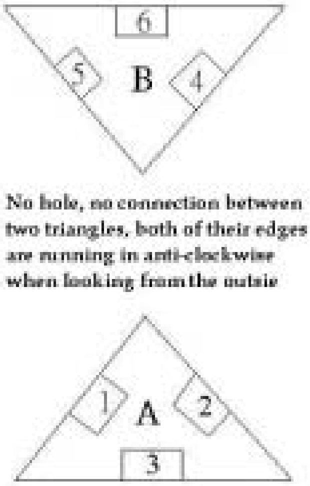


Figure 8 TIN model with two selected triangles



No hole, no connection between two triangles, both of their edges are running in anti-clockwise when looking from the outsie

Figure 9 Edges of triangle A and B

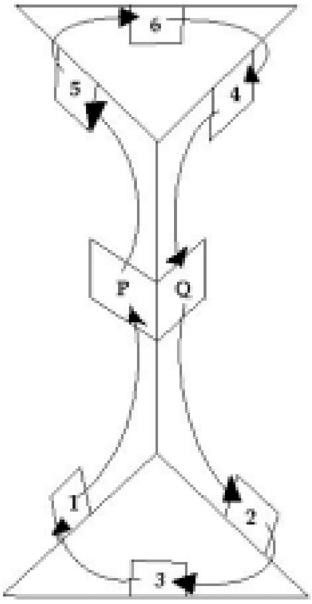


Figure 10 MEHKF

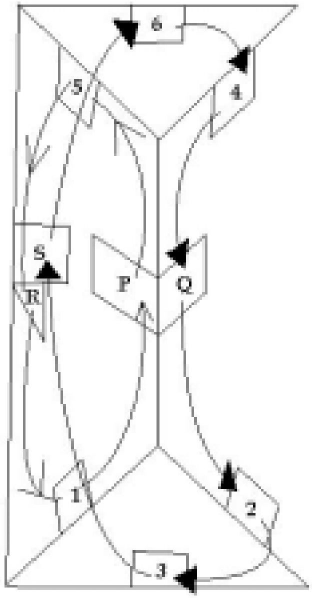


Figure 11 Second step MEF

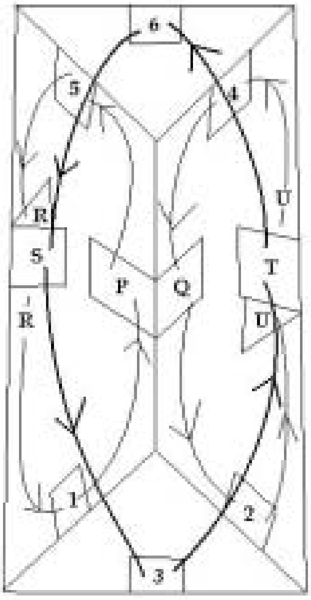


Figure 12 Last step of MEF

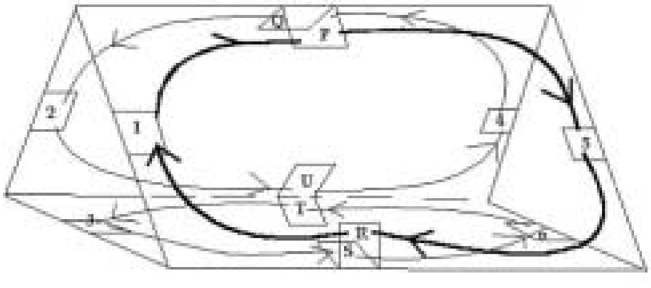


Figure 13 Looking from outside of the hole

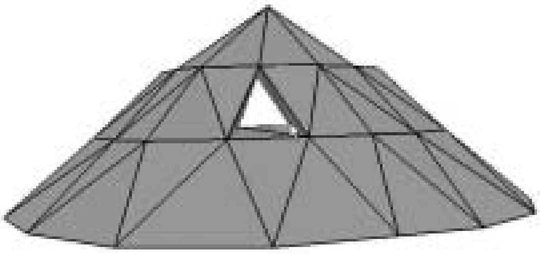


Figure 14 A hole on a TIN model

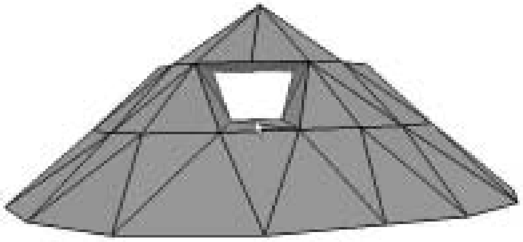


Figure 15 The enlarged hole

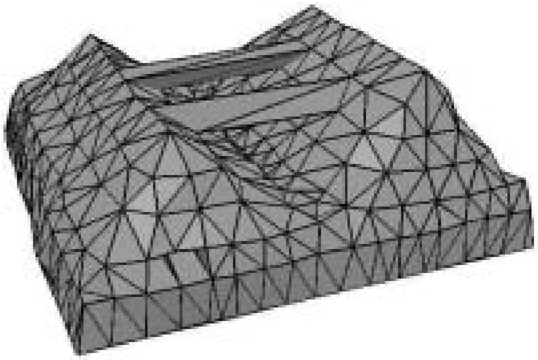


Figure 16 Two bridges on a TIN model

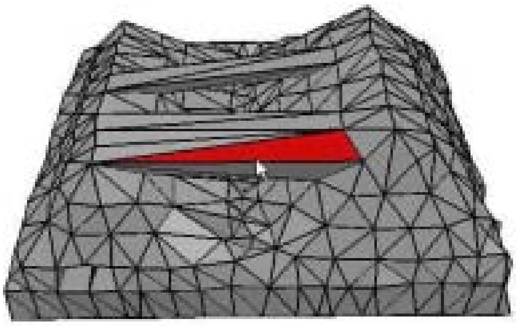


Figure 17 The enlarged bridge

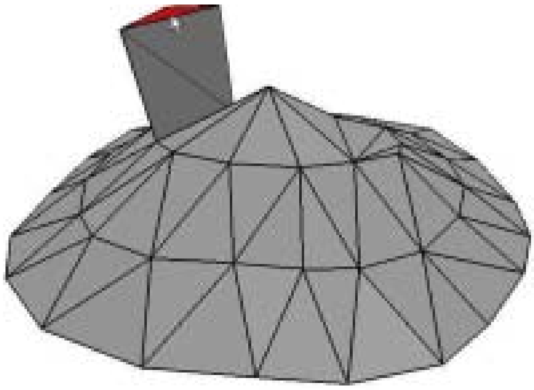


Figure 18 A building on a TIN model

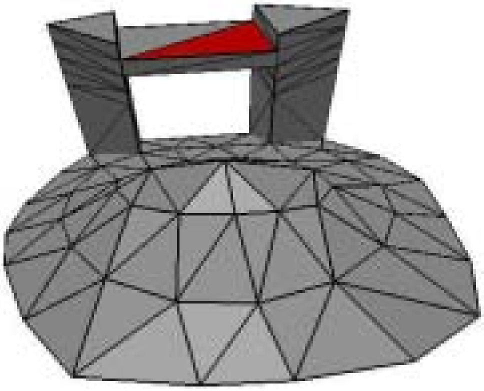


Figure 19 A bridge connects two buildings

9. Conclusion

This article focuses mainly on the technical part of building a 3D cadastral model, and at the same time on the topological relationships between the outside and inside surfaces of complex man-made structures.

In summary, a CAD-based b-rep sense of connectedness is followed. The benefits are:

a) it is developed from the well-known TIN model
b) the addition of the CAD-type properties of Euler operators, including the guarantee of maintaining manifold connectedness, and the addition of features such as holes and buildings
c) a greatly simplified implementation using Quad-Edges rather than the traditional Winged-Edge structure.

The most difficult aspect of maintaining a cadastre is probably maintaining connectedness between spatial elements. This will be much worse in 3D. We believe that a validated b-rep model is a viable extension of current 2D cadastral systems.

Acknowledgements

The authors would like to acknowledge financial support from a research studentship from Hong Kong Polytechnic University, and Research Grant # B-Q488, from the Hong Kong Research Grants Council.

References

Gérard, C. (1999). *Land information systems: services and tools of public land administration*, Chapter 4: Thesaurus on Land Tenure- a FAO/ FIG joint project, France.

Guibas, L. and Stolfi, J., (1985). *Primitives for the implementation of general subdivisions and the computation of Voronoi diagrams.* Transactions on Graphics, v. 4, pp.74-123

Henssen, J., (1995), *Basic principles of the main cadastral systems in the world.* In Proceedings of the One Day Seminar, held during the Annual Meeting of Commission 7, Cadastre and Rural Land Management, of the International Federation of Surveyors (FIG). Delft, The Netherlands, May, 1995.

Kaufmann, J., (1999): *Cadastre 2014: A tool to make land administration more effective?,* FIG99, Sun City, South Africa.

Lee, K., (1999): *Principle of CAD/ CAM/ CAE systems,* Seoul National University, Korea, pp. 101-148.

Mantyla, M., (1981): *Methodological background of the geometric workbench,* Laboratory of Information Processing Science, Otakaari, Finland.

Mantyla, M., (1988): *An introduction to solid modelling,* Computer Science Press, Rockville, MD.

Masuda, H., Shimada, K., Numao, M., and Kawabe, S. (1990) *A Mathematical Theory and Applications of Non-manifold Geometric Modeling*, in Advanced Geometric Modeling for Engineering Applications, North-Holland, Amsterdam.

Weiler, K. (1988) *Boundary Graph Operators for Non-manifold Geometric Modeling Topology Representations,* in Geometric Modeling for CAD Applications, Elsevier Science, Amsterdam.

Williamson, I.P., (1995): *Appropriate cadastral systems*, In Proceedings FIG Commission 7 Annual Meeting, Delft, The Netherlands, May, 1995.

Zeid, I., (1991) *CAD/ CAM theory and practice*, McGraw-Hill, New York, pp368-388.