Improving Accuracy by Incorporating Skeleton Points and Slope Information

Digital Elevation Models from Contour Lines

Despite modern satellite imaging, much topographic data remains in the form of contour lines. For many CIS-supported applications', such as run-off and erosion modelling, contour lines have to be transferred to Digital Elevation Models (DEMs) consisting basically of x, y and z coordinates. The accuracy of slopes, rather than that of heights, is becoming increasingly important in applications such as run-off modelling. The authors examine improved methods for extracting good quality DEMs from topographic contour maps. Gravity models and other methods requiring user-specified parameters should be avoided, Sibson interpolation appears to be the best choice.

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Two features are fundamental in the conversion of contour maps to Digital Elevation Models (DEMs):

- Sampling density along the contours
- Interpolation of the irregular set of data points to a regular grid

Valid theorems for sampling density have only recently been developed. Interpolation methods have to deal with the 'flat triangle' problem, as well as preservation of the accuracy of slope. Meaningful slopes are increasingly important in applications such as soil erosion and run-off modelling. The 'flat triangle' problem appears when the vertices of triangles have the same elevation which, for example, occurs at ridges and valleys. The problem is solved by generating a medial axis transform, or skeleton, between the curves.

To arrive at DEMs with accurate slopes in which the 'flat triangle' problem is solved, we propose the following approach:

- Generate skeleton points and ignore skeletons between contours
- Assign elevations to these skeleton points
- Eliminate 'flat triangles' by- the insertion of these skeleton points into the original dataset
- Estimate slope information at each data point
- Perform weighted-average interpolation using the estimated slope information

The approach is described and examined below, with the assumption that the data is sufficiently well sampled.

Voronoi Diagram and Delaunay Triangulation

Our approach depends on the Voronoi diagram and its dual, Delaunay triangulation (Figure la). The first is often used to partition a map into regions closest to each generating point. Delaunay triangulation is usually used as the basis for triangulating a set of data points, as it is guaranteed to be locally stable. It may easily be constructed using its property of the 'empty circumcircle'. This circle is centred at the Voronoi node associated with each triangle and

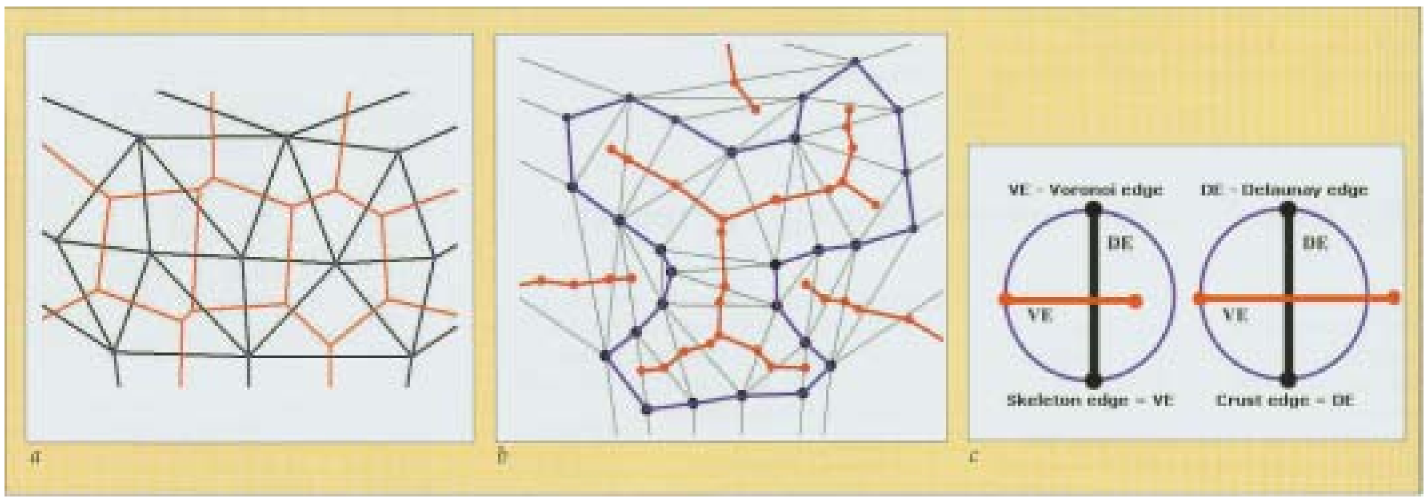

Figure 1, (a) Delaunay triangulation (black lines) and Voronoi diagram (red lines) (b) crust (blue edges) and skeleton (red edges) (c) Crust/skeleton test passes through the three vertices. The Voronoi diagram and Delaunay triangulation are associated with the 'crust' and the 'skeleton' (Figure 1b). The crust is formed from the triangle edges which do not cross the skeleton. If the sampling of the curve is less than 0.25 of the distance to the skeleton formed by the remaining Voronoi edges, the crust is guaranteed to be correct. Gold and Snoeyink simplified this property, noticed by Amenta and co-workers, by showing that in every Delaunay/ Voronoi edge pair either the Delaunay edge may be assigned to the crust or else the dual Voronoi edge can be assigned to the skeleton. The Delaunay edge belongs to the crust when there exists a circle through its two vertices that contain neither of its associated Voronoi vertices; otherwise the corresponding Voronoi edge belongs to the skeleton. A simple, well-known InCircle test applied to each Delaunay/ Voronoi edge pair distinguishes these cases (Figure 1c).

Slopes important for soil erosion modelling

Ridge and Valley

Figure 2a shows a close-up of the test set with shaded flat triangles. In a Delaunay triangulation the circumcircles may not contain any data points. With the insertion of a skeleton point in a flat triangle, the triangle has to be replaced by new triangles with the skeleton point as a vertex. We introduce two interpolation techniques to estimate skeleton point elevations before their insertion. Both are here briefly explained for the case of a ridge or valley, a simple case of the 'flat triangle' problem. In the first method, a constant side-wall slope is assumed and skeleton heights are estimated using circumradius ratios (Figure 2b). The larger circle, at the junction of the skeleton branches, has a known elevation -halfway between the contours -and is used to determine the local slope. The elevation of the centre of the smaller circle is thus based on the ratio of the two radii. In the second approach, where constant slope down the valley is assumed, the line of the valley is determined by searching along the skeleton and heights are assigned based on their relative distance along this line. In the results below. Figures 3 to 5 use the valley length, while Figure 6 uses constant side-wall slope.

Interpolation Method

A weighted-average interpolation method consists of three main components:

- The weighting model
- The set of neighbours used to obtain the average
- The elevation function being averaged; often one uses the data alone. However, we define a planar function through each data point involving its height and local slopes. In this way, a set of z values at any grid node is calculated by inserting the (x, y) of the grid node into each of the functions of the neighbouring data points. These z estimates are then weighted and averaged

We use three weighted-average interpolation methods:

- Triangle-based interpolation. This is one of the simplest models: a linear interpolation is performed within each triangle. Figure 3a shows the resulting surface, in which flat triangles are readily seen. Figure 3b shows the improvement when estimated skeleton points are added and all flat triangles are automatically removed
- The gravity model. The weighting is inversely proportional to the square of the distance from the data point to the grid node (other exponents may also be used). The data points involved are determined by the setting of a radius with the grid node as centre. Figure 4a shows the resulting surface for a radius of about a quarter of the map. The result depends on the radius and the setting of the exponent. With too small a radius, holes (empty circles with no data points) may be present. Increase in the radius may cause flattening of the

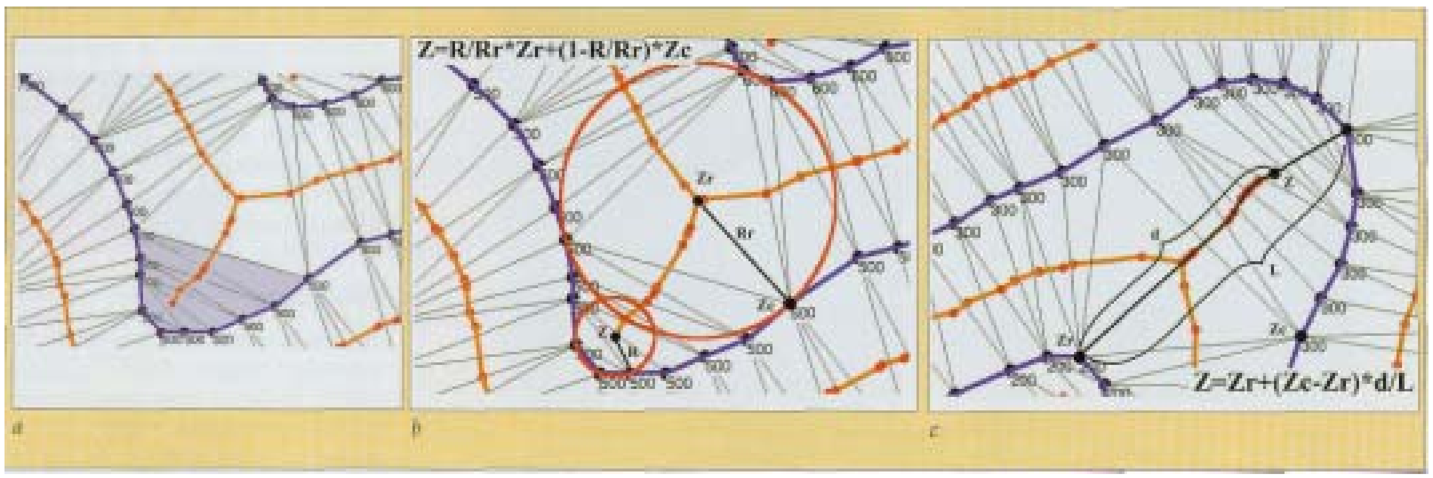

Figure 2, (a) Skeleton and 'flat triangles' (b) Estimating skeleton heights from circumradii (c) Estimating skeleton heights from valley length

Figure 3, (a) Triangle-based interpolation, (b) Adding skeleton points

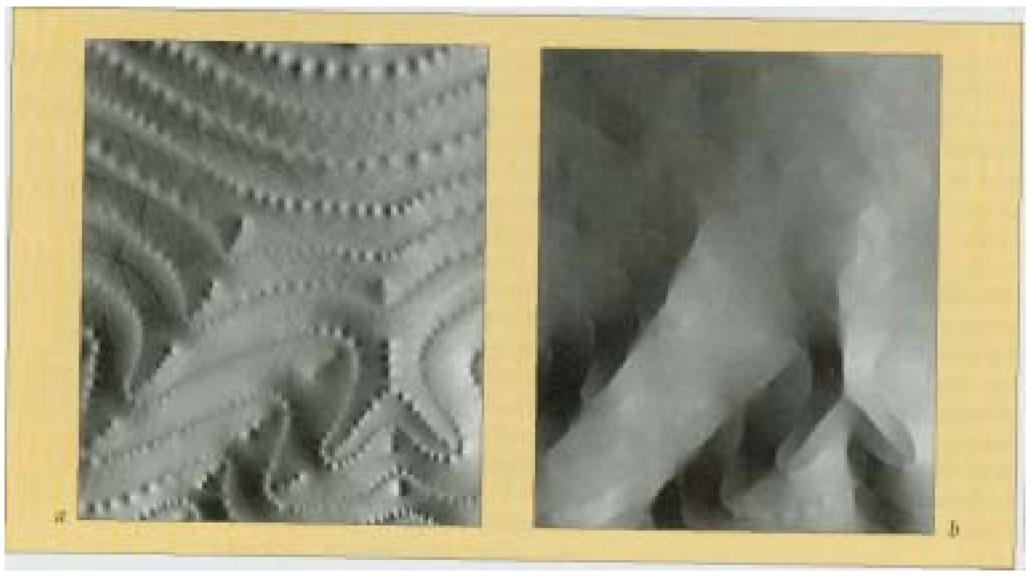

***Figure** 4, (a) Interpolation using the gravity model with medium radius, (b) Sibson interpolation*

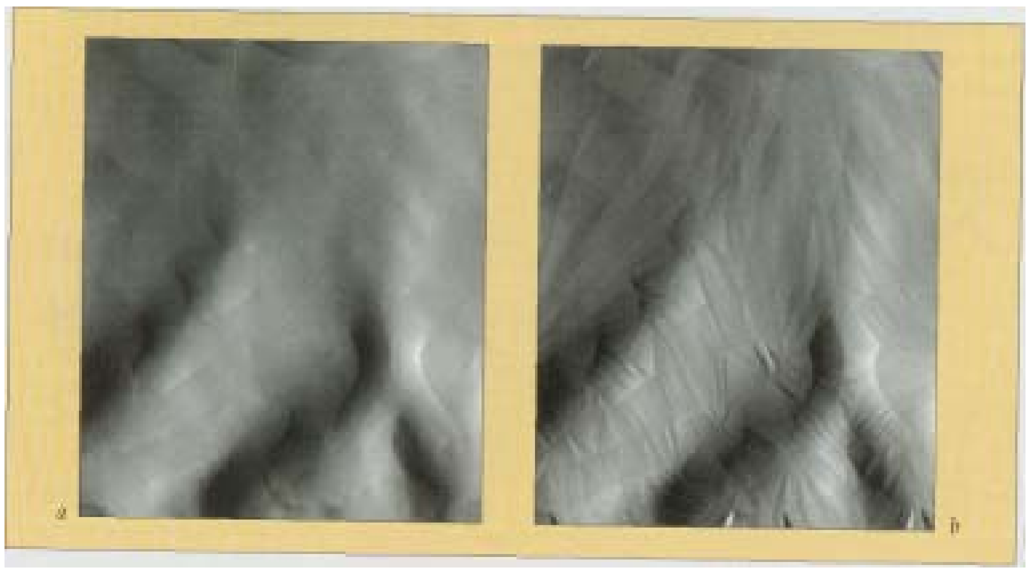

***Figure** 5, (a) Sibson interpolation using slopes at data points, (b) adding slopes at data points for triangle-based interpolation*

surface. When the data distribution is highly anisotropic considerable difficulty may be encountered in the selection of a valid counting circle radius

- The Sibson method; each grid node is temporarily inserted into the Voronoi diagram. The number of data points used is determined automatically. Their Voronoi-areas determine the weighting. This method is particularly appropriate for poor data distributions. Figure 4b shows the results of Sibson interpolation. The surface behaves well but is angular at ridges and valleys. Indeed, Sibson states that slopes are discontinuous at all data points

Results

Vertical views were generated using version 5 of the Manifold GIS, available from www.Manifold.net; 3D visualisation has been under-utilised as a tool for testing terrain modelling algorithms and the results are often more useful than a purely mathematical, or even statistical, approach. Figure 5a shows the result of using Sibson interpolation with data point slopes. The form is good, but slight breaks in slope can be seen at contour lines. Adding slopes using the position in the triangle to provide the weights, as in Figure 5b, produces results that are almost as good as the Sibson method when the density of the sample points is high. However, the Sibson method is much superior for sparser data or when the points do not form contour lines. The gravity model does not provide particularly good slope estimates but, even here, including the data point slope function produces a significant improvement. Figure 6, constructed as an artificial example of four small hills defined by their contours, shows clearly that Sibson interpolation is superior to simple triangle-based interpolation.

Conclusions

To produce good DEMs with reasonable slopes from contour maps, skeleton points with estimated elevations have to be added in order to eliminate flat triangles. The addition of slope information at data points and its

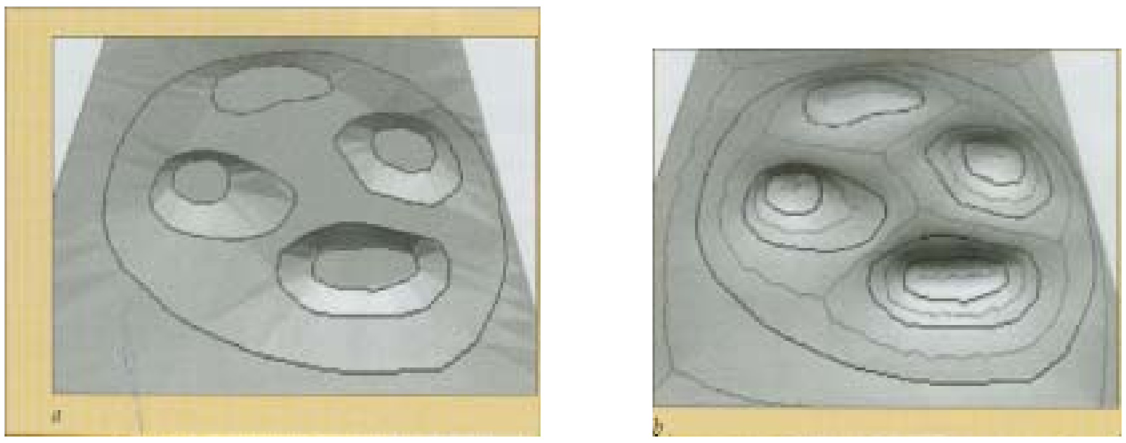

Figure 6, *Triangulation of several small hills, (a) triangle-based interpolation (b) Sibson interpolation with slopes*

use in the weighted average interpolation process significantly improves even poor interpolation methods. Also important is the selection of a meaningful set of data points around the estimated point. Of lesser importance is the particular interpolation method used, although this statement is highly dependent on data distribution and density. Gravity models and other methods requiring user-specified parameters should be avoided. Sibson interpolation appears to be the best choice. Surprisingly mathematically guaranteed slope continuity is not usually critical. For the reconstruction of surfaces from contours and the generation of DEMs our approach appears to represent a significant improvement on previous work.

Acknowledgements

A grant from the Hong Kong Polytechnic University (Project No. PolyU A-PB79) is kindly acknowledged.

Further Reading

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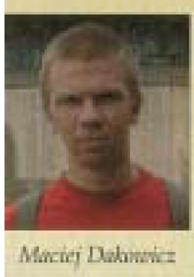
Biography of the Authors

Christopher Gold is professor in the Department of Land Surveying and Geo-Informatics, The Hong Kong Polytechnic. His PhD concerned spatial modelling in geology. He first worked with TIN models in the 1970s. More recently, his attention has been directed towards Voronoi diagrams and he has developed a variety of applications. More information on his work and on Voronoi diagrams may be found on www.Voronoi.com.

Maciej Dakowicz is a research associate in the same department. He received an MSc degree in Computer Science from Bialystok Technical University, Poland in 2000. The same year he moved to Hong Kong to work on Voronoi diagrams. His specialty is terrain modelling and topological data structures



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